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Time As a Geometric Property of Space

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The proper description of time remains a key unsolved problem in science. Newton conceived of time as absolute and universal which *"flows equably without relation to anything external."* In the nineteenth century, the four-dimensional algebraic structure of the quaternions developed by Hamilton, inspired him to suggest that he could provide a unified representation of space and time. With the publishing of Einstein's theory of special relativity these ideas then lead to the generally accepted Minkowski spacetime formulation of 1908. Minkowski, though, rejected the formalism of quaternions suggested by Hamilton and adopted an approach using four-vectors. The Minkowski framework is indeed found to provide a versatile formalism for describing the relationship between space and time in accordance with Einstein's relativistic principles, but nevertheless fails to provide more fundamental insights into the nature of time itself. In order to answer this question we begin by exploring the geometric properties of three-dimensional space that we model using Clifford geometric algebra, which is found to contain sufficient complexity to provide a natural description of spacetime. This description using Clifford algebra is found to provide a natural alternative to the Minkowski formulation as well as providing new insights into the nature of time. Our main result is that time is the scalar component of a Clifford space and can be viewed as an intrinsic geometric property of three-dimensional space without the need for the specific addition of a fourth dimension.

Keywords: time, geometric algebra, quaternions, Minkowski, spacetime

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1. INTRODUCTION

Historically, there have been many attempts to understand the nature of time and provide a rigorous definition. One of the most influential ideas regarding time was published in 1686 by Sir Isaac Newton, *Principia* Book 1, *"Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, ..."* [1]. Newton's apparent conceptualization of a universal and perfect clock is significantly modified by relativity theory, which finds that the observed rate of clocks depends on relative motion as well as the relative strength of the gravitational field, between source and observer. So, in order to retain Newton's definition, we are required to add the proviso that it applies to an inertial observer in the rest frame of the system under observation, and to a region of space near the observer so that differential gravitational fields become negligible. So in this frame of reference, exactly what sort of clock can we then utilize? One possible candidate is the de Broglie frequency that is assumed to be associated with every particle of energy E, which in its own rest frame, is given by $\nu = E/h = mc^2/h$, where m is the invariant mass of the particle and h is Planck's constant[1]. We would expect observers throughout the universe observing fundamental particles, such as electrons, in this way, would all observe the same frequency and thus provide a universal clock. Observations of distant cosmic objects appear to confirm that the laws of nature and the fundamental constants are essentially invariant throughout the universe, and only some small variations have been claimed [3].

We can also attempt to draw further insight from the representation of time in the Minkowski spacetime framework $[t, x, y, z]$, where t is the time coordinate. This formulation appears to imply that time is a dimension, orthogonal to the other three space dimensions and so naturally leads to the concept of time being a fourth dimension. A limitation on the idea of treating time as a fourth Euclidean-type dimension, though, is that it does not appear possible, at least in the macroscopic world, to freely travel in the time dimension as is possible with space dimensions [4]. Hence, the Minkowski formulation seems somewhat unhelpful in clarifying this aspect of time [5].

1.1. Algebraic Formulations of time

In order to proceed with our approach, we now wish to algebraically model three-dimensional physical space. Unfortunately, it appears, that despite over a 100 years of intense development there is still no generally accepted algebraic description of space. That is, distinct approaches are commonly utilized to model various physical systems such as three-vectors, four-vectors, matrices, complex numbers, quaternions and complex spinors, depending on the specific application. This fact is indeed quite surprising, in that, one of the specific goals of nineteenth century science was to find the correct algebra for physical space [6]. The objective was initially led by Hamilton who produced the quaternion algebra through generalizing the two-dimensional complex numbers to three dimensions. Indeed, due to the general success of complex numbers in algebraically describing the properties of the plane Hamilton reasoned that quaternions should therefore properly describe the algebra of three-dimensional space. This then lead to the first attempt at a rigorous mathematical definition of a unified space and time. Hamilton wrote the quaternion as

$$q = t + x_1\boldsymbol{i} + x_2\boldsymbol{j} + x_3\boldsymbol{k}, \tag{1}$$

where $t, x_1, x_2, x_3 \in \Re$ and the three basis vectors are subject to the well known quaternionic relations $\boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1$. Hamilton's quaternions form a four-dimensional associative normed division algebra over the real numbers represented by $\mathbb{H}$. Hamilton defined $\boldsymbol{x} = x_1\boldsymbol{i} + x_2\boldsymbol{j} + x_3\boldsymbol{k}$ as a vector quaternion to take the role of Cartesian vectors in order to describe spatial vectors, and then also proposed, around 50 years before Minkowski, that if the scalar "t" was identified with time then the quaternion can be used as a representation for a unified four-dimensional spacetime. Hamilton stating *"Time is said to have only one dimension, and space to have three dimensions. ... The mathematical quaternion partakes of both these elements; in technical language it may be said to be 'time plus space', or 'space plus time': and in this sense it has, or at least involves a reference to, four dimensions"* [7]. Indeed squaring the quaternion we find

$$q^2 = t^2 - \boldsymbol{x}^2 + 2t\boldsymbol{x} \tag{2}$$

the scalar component $t^2 - \boldsymbol{x}^2$ thus producing the invariant spacetime distance.

Minkowski, after indeed considering the quaternions, but viewing them as too restrictive for describing spacetime, chose rather to extend the Gibbs-Heaviside three-vector system with the addition of a time coordinate to create a four-component vector producing the modern description of spacetime, as a four-vector

$$X = [t, \boldsymbol{x}]. \tag{3}$$

Defining an involution $\bar{X} = [t, -\boldsymbol{x}]$ we then produce the invariant distance

$$X \cdot \bar{X} = [t, \boldsymbol{x}] \cdot [t, -\boldsymbol{x}] = t^2 - \boldsymbol{x}^2, \tag{4}$$

as required [8].

Comparing these two descriptions we can see that they provide some significant differences. To begin with, Hamilton's description views time as an intrinsic part of the description of three-dimensional space. On the other hand, with the Minkowski formulation, the immediate implication is that time is an additional Euclidean-type dimension. Although this assumption is qualified by the fact that time contributes an opposite sign to the metric distance and so distinct from a regular four-dimensional Cartesian vector. Note that the octonions, being the generalization of quaternions, have also been considered as an expanded arena for spacetime [9–12].

It is an historical fact that the vector quaternions were found difficult to work with and not suitable to describe Cartesian vectors and were replaced by the Gibbs vector system in use today [6]. The reason for Hamilton's failed attempt to algebraically describe three-dimensional space, is that by generalizing the complex numbers of the plane to three dimensions he actually only produced the rotational algebra for three dimensions. To properly describe three-dimensional space we further need to generalize the quaternions to include a true Cartesian vector component [13]. This generalization of quaternions was in fact achieved by Clifford with the eight-dimensional Clifford algebra over three dimensions of $C\ell(\Re^3)$.

2. RESULTS—CLIFFORD'S DESCRIPTION OF SPACE

A Clifford geometric algebra $C\ell(\Re^n)$ defines an associative real algebra over n dimensions and in three dimensions $C\ell(\Re^3)$ is eight-dimensional [13, 14]. In this case we can adopt the three quantities e_1, e_2, e_3 for basis vectors that are defined to anticommute in the same way as Hamilton's quaternions, but unlike the quaternions these quantities square to positive one,

[1]The de Broglie frequency can also be associated with the *Zitterbewegung* frequency predicted by Schrödinger in 1930 and now confirmed experimentally [2].

that is $e_1^2 = e_2^2 = e_3^2 = 1$. Also, similar to quaternions, we can combine scalars and the various algebraic components into a single number, called in this case a multivector

$$X = t + x_1 e_1 + x_2 e_2 + x_3 e_3 + n_1 e_2 e_3 + n_2 e_3 e_1 + n_3 e_1 e_2 + b e_1 e_2 e_3, \tag{5}$$

where $t, x_1, x_2, x_3, n_1, n_2, n_3, b \in \Re$. Now defining $j = e_1 e_2 e_3$ we find the dual relations $je_1 = e_2 e_3$, $je_2 = e_3 e_1$ and $je_3 = e_1 e_2$, which allows us to write

$$X = t + \boldsymbol{x} + j\boldsymbol{n} + jb, \tag{6}$$

with the vectors $\boldsymbol{x} = x_1 e_1 + x_2 e_2 + x_3 e_3$ and $\boldsymbol{n} = n_1 e_1 + n_2 e_2 + n_3 e_3$.

It can be shown that quaternions are isomorphic to the even subalgebra of the multivector, with the mapping $\boldsymbol{i} \leftrightarrow e_2 e_3$, $\boldsymbol{j} \leftrightarrow e_1 e_3$, $\boldsymbol{k} \leftrightarrow e_1 e_2$ and the Gibbs vector can be replaced by the vector component of the multivector. Thus, within Clifford's system we can absorb the quaternion as $q = t + j\boldsymbol{n}$ and a Gibbs vector x into a unified system [14].

We define *Clifford conjugation* on a multivector X as

$$\bar{X} = t - \boldsymbol{x} - j\boldsymbol{n} + jb. \tag{7}$$

We define the *amplitude* squared of a multivector X as $|X|^2 = X\bar{X}$ that gives

$$|X|^2 = t^2 - \boldsymbol{x}^2 + \boldsymbol{n}^2 - b^2 + 2j\left(tb - \boldsymbol{x} \cdot \boldsymbol{n}\right) \tag{8}$$

forming a commuting "complex-like"[2] number.

Clifford conjugation that produces the multivector amplitude turns out to be the only viable definition for the metric as it is the only option that produces a commuting resultant and is thus an element of the center of the algebra. This is essential as we require the metric distances to be isotropic in space in order to be generally consistent with the principles of relativity. Once again we can observe the required invariant distance $t^2 - \boldsymbol{x}^2$ appearing in the metric.

An important point to note for the multivector, is that in order to produce a meaningful metric, which consists of a combination of its various elements, then all of these components must be measured in the same units, as shown in Equation (8). Now, beginning from 1983, the General Conference on Weights and Measures (CGPM) decided that the speed of light should be assumed constant and that a meter was then the distance traveled by light in a specified time interval equal to $1/c$ s [15]. The CGPM defines the speed of light as a universal constant $c = 299,792,458$ m/s. Hence both time and distance are now measured in units of seconds, and it is therefore natural to adopt these units for all components of the multivector. Distances typically measured in meters therefore appear in the multivector with the conversion $\boldsymbol{x} \rightarrow \boldsymbol{x}/c$ and so are in units of seconds. Interestingly, from the perspective of the multivector, c is simply taking the role of a units conversion factor, and so therefore the value of c is obviously invariant between observers. Hence one confusion regarding time could arise due to the poor selection of units, as in order to properly relate space and time they should be measured in the same units, as now carried out by CGPM.

The role of the various terms in the metric can be understood from several perspectives. Geometrically they refer to the four geometric elements of three-dimensional space (points, lines, areas and volumes) combined in a natural way to form an invariant distance. These four geometric elements described algebraically by the scalar, vectors, bivectors and trivectors of Clifford geometric algebra. In terms of physical quantities they are commonly referred to as scalars, vectors, pseudovectors and pseudoscalars. Now, scalars and vectors are well understood physically as quantities such as energy and momentum respectively. Pseudovectors, also called axial vectors, can be understood physically as rotational quantities, such as angular momentum, torque and the magnetic field. Pseudoscalars are less commonly understood and describe the nature of magnetic monopoles or helical motion.

The multivector generalization of the quaternions now allows a full algebraic description of three-dimensional space, as the scalar, vector, bivectors and trivectors components now correspond directly with the geometrical quantities of points, lines, areas and volumes found in three dimensions. Additionally these four quantities describe the range of physical quantities described as scalars, vectors, pseudovectors (or axial vectors) and pseudoscalars [13]. Now, similar to quaternions, as we also identify time as the scalar part of space, time is now imputed the geometrical meaning and topology of a scalar point-like quantity. This can be contrasted with the linear description of space as vectors implied by Minkowski and so we can see now a sharp geometrical distinction between space and time when described within the multivector. We now wish to show briefly that the multivector provides a viable formalism to describe both classical and relativistic dynamics from which a definition of time can then arise.

2.1. Classical and Relativistic Mechanics

The amplitude of a multivector defined in Equation (8) effectively defines distances in the Clifford representation of space. Hence, we can define the action in the conventional manner between two spacetime locations, represented by multivectors, as

$$S = \int |dX|. \tag{9}$$

Now, if we ignore the imaginary like components, for simplicity, we have

$$|dX|^2 = \left(\dot{t}^2 - \dot{\boldsymbol{x}}^2 + \dot{\boldsymbol{n}}^2 - \dot{b}^2\right) d\tau^2, \tag{10}$$

where we define $\dot{t} = \frac{dt}{d\tau}$, $\dot{\boldsymbol{x}} = \frac{d\boldsymbol{x}}{d\tau}$, $\dot{\boldsymbol{n}} = \frac{d\boldsymbol{n}}{d\tau}$ and $\dot{b} = \frac{db}{d\tau}$. The symbol τ is a scalar that can be identified with the scalar time t when the other variations within the multivector are zero, that is the time in the rest frame of the particle, commonly referred to as the proper time. Hence we can see that no external time parameter is required in this approach as time arises from within the spacetime multivector itself and with the assumption of a rest

[2] We refer to this as a complex-like number because the trivector j is commuting and squares to minus one and all other quantities are real scalars.

frame a suitable evolution parameter can be identified. If we write the action $S = \int \frac{|dX|}{d\tau} d\tau$ then this implies a Lagrangian

$$\mathcal{L} = \frac{|dX|}{d\tau} = \sqrt{\dot{t}^2 - \dot{\boldsymbol{x}}^2 + \dot{\boldsymbol{n}}^2 - \dot{b}^2} = 1, \qquad (11)$$

where we now extremize the action $S = \int \mathcal{L} d\tau$.

As we have no explicit coordinate dependence, $\frac{\partial \mathcal{L}}{\partial t}$, $\frac{\partial \mathcal{L}}{\partial x}$, $\frac{\partial \mathcal{L}}{\partial n}$ and $\frac{\partial \mathcal{L}}{\partial b}$ are constants of the motion. Using the Euler-Lagrange equation for t

$$\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{t}} - \frac{\partial \mathcal{L}}{\partial t} = 0 \qquad (12)$$

thus giving the conserved quantity

$$\frac{\partial \mathcal{L}}{\partial \dot{t}} = \mathcal{L}^{-1}\dot{t} = E. \qquad (13)$$

We have written the conserved quantity as the dimensionless scalar E as we expect it to relate to energy by Noether's theorem [16]. Indeed, because $\dot{t} = dt/d\tau = \gamma$ and $\mathcal{L}^{-1} = 1$, we find $E = \gamma$, which is the conventional relativistic energy relation for a unit mass. That is, inserting the speed of light factor and an invariant scalar mass factor we find $E = \gamma mc^2$ for the energy, as required. Also, using the Euler-Lagrange equation for x we find the conserved quantity

$$\frac{\partial \mathcal{L}}{\partial \dot{\boldsymbol{x}}} = \mathcal{L}^{-1}\dot{\boldsymbol{x}} = p. \qquad (14)$$

Thus the second conserved quantity is the momentum per unit mass $\boldsymbol{p} = \gamma \boldsymbol{v}$, where $\boldsymbol{v} = \frac{d\boldsymbol{x}}{dt}$. The bivector and trivector components will relate to the conservation of angular momentum and spin. That is, we will produce the conserved quantities $\boldsymbol{\ell} = \gamma \boldsymbol{w}$, where $\boldsymbol{w} = \frac{d\boldsymbol{n}}{dt}$ and $s = \gamma \frac{db}{dt}$. Hence physical space defined using the multivector in $C\ell(\Re^3)$, with the metric defined by Equation (8), forms a spacetime naturally producing the four conventional conservation laws for motion.

For completeness we note that transforming the multivector as $X' = AXB$ where $A, B \in C\ell(\Re^3)$ will leave the metric distance defined in Equation (8) unchanged, and so this transformation defines the Lorentz transformations. Due to the higher eight-dimensional nature of spacetime that we have assumed using the multivector these will obviously form a generalization of the conventional Lorentz transformations. Note though that in the special case of four-dimensional spacetime events we reproduce the conventional six-dimensional Lorentz group.

The first two components of the multivector $\dot{t} + \dot{\boldsymbol{x}} = (E + \boldsymbol{p})/m$ can be identified as the four-momentum per unit mass. The third and fourth components $j\dot{\boldsymbol{n}} + j\dot{b}$ has the transformational properties of four-spin. Hence for a general particle with momentum and spin we produced a unified description. Hence the metric in Equation (8) provides an invariant distance combining these two measures. This particular distance measure has the critical properties of producing the expected Minkowski spacetime distance as well producing a commuting value and being one of the simplest possible. Because it is commuting, all observers agree on this measure, assuming the bilinear tranformation between inertial observers. Interestingly this invariant distance, in general, has an imaginary component, and represents a further conserved quantity between observers.

Note that we have assumed a rest frame for the particle, so that we could form the proper time τ as an evolution parameter. However this is not always possible for particles such as photons for example, and so we need to consider time somewhat more generally than the scalar component alone. That is, we need to look more broadly at the full multivector in order to more completely define time.

3. DISCUSSION

In order to aid our intuition, we can use a rubber sheet analogy to give a geometrical picture of the multivector and hence a more tangible understanding of space and time. The scalar component t can represent the expansion or contraction of the sheet at each point, the displacement x the linear distortion within the sheet, the bivector jn the planar twist and the trivector jb the three-dimensional torsion of the sheet. It is interesting that the scalar can refer to the expansion of the rubber sheet, and so can form a correspondence with cosmic time measured with the expansion of the universe. This identification of time with expansion is also consistent with Newton's concept of the non-directional flow of time with space expanding uniformly in all directions.

The Clifford multivector description of spacetime views time and space as simply two particular geometrical properties that we abstract from three-dimensional space with the scalar component representing time. Note that this definition of time as the scalar element implies that time can be applied to the general space $C\ell(\Re^3)$ of general dimension n. It is also natural to ask then regarding the other two geometrical components of the multivector, that of bivectors and trivectors. These of course we refer to commonly as area and volume but also refer to physical quantities having the attributes of pseudovectors and pseudoscalars, respectively. Inspecting the metric in Equation (8) we can see that the bivector component also provides the correct signature for the time component as well as the scalar component that we initially selected, giving the modified metric for the spacetime interval of $(dt^2 + dn^2) - dx^2$, ignoring for the present discussion the other contributions to the metric. This appears to indicate that the whole quaternion $T = t + j\boldsymbol{n}$ should therefore be used to represent the property of time. This makes sense as it then unifies the two key aspects of time that we have identified that of the reversible rotational part jn and the irreversible expanding part as the scalar t. If this line of thinking is adopted then time becomes a four-dimensional quaternionic quantity. Indeed multi-time theories of spacetime have already been investigated [17–22]. As an intriguing side-note, Hamilton specifically referred to the quaternionic algebra he developed as the "algebra of pure time" [23].

Time is actually closely linked with rotation. For example, Kepler's second law states that for gravitationally bound orbits, or indeed any central force field, that a satellite's position vector will trace out 'equal areas in equal times' or $dA = kdt$, where

k is some constant. This relation can thus be inverted to define a precise and steady clock $dt = (1/k)dA$ and a definition of time consistent with a bivector. This heuristic picture using Kepler's laws is not meant to be precise, however, it does show a connection between time and rotation. More specifically we know that bivectors generate rotations in a Clifford space and indeed the unit quaternions are commonly used to describe rotations in three dimensional space.

The fourth and final geometrical component of physical space is the trivector jb that describes the torsion of physical space or helical motion of particles typified, for example, by a circularly polarized photon. It should be noted though that this term has a space-like contribution to the metric and so we would not include it as an aspect of time.

An important implication of the metric distance in spacetime being defined by Equation (8) is that by measuring distance exclusively based on the scalar and vector contributions to the metric distance, as is typically done, we are not calculating distances and times in a fully invariant manner.

The Clifford algebra $C\ell(\Re^3)$ has several subalgebras including the real scalar, the quaternions $t + j\boldsymbol{n}$ as well as the complex numbers $t + jb$. Also, while not closed, the spaces $t + \boldsymbol{x}$ describe conventional spacetime while $j\boldsymbol{n} + jb$ can describe four-spin. Typically the energy and momentum of massive particle is represented by the four-vector $[E, \boldsymbol{p}]$. However, this obviously ignores the spin-angular momentum attributes of fundamental particles represented as $j\boldsymbol{l} + js$. Hence it is natural to combine these description into a full momentum multivector as $P = E + \boldsymbol{p} + j\boldsymbol{l} + js = mdX/d\tau$, where m is the invariant mass and using the space multivector X defined in Equation (6). Also note that $\boldsymbol{E} + j\boldsymbol{B}$ can describe the electromagnetic field.

4. CONUNDRUMS REGARDING TIME

1. Time is generally considered as being an ineffable concept essentially invisible to the senses—in comparison, space appears much more tangible and visualizable.

This distinction between time and space, can perhaps be traced back to their distinct topological natures as a point-like and line-like quantities respectively that we have identified. A true point, by definition, is invisible to observation.

2. Why it is possible to freely move in space but not in time?

The Minkowski formulation, in which time and space are combined within a four-vector, leads one to think that we should be able to move in the time dimension as freely as we move in the space dimension. Now, if we take a coffee cup as an example, then clearly we can indeed move it in a certain direction and then easily reverse this translation by moving it back again to its starting point. For rotating the cup it appears that we can do the same, however, if we consider time as related to the microscopic rotations at the fundamental particle level and all the spin axes are randomly aligned then it is not possible to reverse all these microscopic spin directions simultaneously. This is distinct from the space direction due to the molecular bonding of the atoms where it is indeed possible to reverse the spatial movement of each constituent particle simultaneously. However, when we move to the level of fundamental particles the rotational nature of time allows fundamental particles to move backwards in time if they invert their rotation, as in the CPT symmetry.

3. Why we tend to perceive an arrow of time?

If we describe time by the quaternion $T = t + j\boldsymbol{n}$, then we can produce both a irreversible (scalar) and a reversible (bivector) aspect to time.

5. CONCLUSION

We began from the premise that the eight-dimensional Clifford geometric algebra $C\ell(\Re^3)$ provides an appropriate algebraic description of three-dimensional physical space. The multivector naturally describes algebraically the four geometric elements of three-dimensional space, that of points, lines, areas and volumes, as shown in Equation (6). These four quantities also describe the physical quantities referred to as scalar, vectors, pseudovectors and pseudoscalars and found to be a natural language to describe physical theories in three dimensions [24, 25]. We thus consider that the four geometrical quantities of physical space are the fundamental basis upon which we abstract such local concepts as time and space [26, 27]. When we represent spacetime with the Clifford multivector, in order to be consistent with Minkowski spacetime we find that time needs to be identified with the scalar component and space with the vector component of the multivector. This thus gives a view of time as the geometric point-like quantities of space. With vectors describing space, we thus now have a geometrical union of time and space as the points and lines, respectively. Thus the correct topology of time can be proposed as a point-like entity that is distinct from the Minkowski formulation that implies a linear topology [28]. Also, using the rubber sheet analogy we noted that the scalar can represent an expansion of the sheet and so nicely correlated with cosmic time as defined by an expanding universe. This idea of time represented by expansion also neatly ties in with the Newtonian idea of the non-directional flow of time and the principle of entropy. We also wish to note that the bivectors are also time-like and so could be included as an additional description of time. Thus time would effectively become a four-dimensional quaternionic quantity, combining both scalar and bivector components. This idea could be further explored as it may enable us to unify two different attributes of time, with the scalar attribute representing the irreversible expansional aspect of time and the bivector the reversible rotational attribute of time, respectively.

So "what is time?" We conclude that it is simply one of the properties that we abstract from the geometry of the three-dimensional space, primarily the geometrical point-like quantities as well as the areal bivector quantities.

We conclude that the Minkowski representation of spacetime, which rejected the quaternions in favor of the four-vector formalism, was a move away from the more natural geometrical description of time using Clifford algebra $C\ell(\Re^3)$.

Ultimately the full union of time and space needs to include all the geometric elements of point-like, linear, areal and volume elements and so implies that a four-vector is insufficient, but that we require an eight-dimensional spacetime event multivector, incorporating the scalar, linear, rotational and the torsional aspects of space, as shown in Equation (6). In addition to the scalar and vector components of the multivector being labeled as time and space respectively, we propose that the additional bivector time-like component and the trivector space-like component should also be properly identified and labeled. The fact that these additional components within spacetime are generally ignored could help explain some of our current difficulties in properly understanding time.

In conclusion, if we assume that the correct algebraic representation of physical space is given by Clifford geometric algebra, then the Minkowski metric implies that time is the scalar aspect of this space, represented geometrically as points. This then implies a Lagrangian leading to relativistic mechanics and by extension the other main laws of physics and is hence a meaningful and consistent description of time.

AUTHOR CONTRIBUTIONS

JC formulated the original conception. All authors made subsequent critical revisions, interpretation, and proof read the manuscript. All authors gave final approval of the version to be published and agree to be accountable for all aspects of the work.

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