Can the $\Lambda\pi$ scattering phase shifts be large?

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Motivated by the presence of nearby thresholds in other baryon-meson channels with $I=1$ and $S=-1$, we investigate whether the $\Lambda\pi$ scattering phase shifts at a center-of-mass energy equal to the Ξ mass could be larger than suggested by lowest-order chiral perturbation theory. Within a coupled-channel K-matrix approach, we find that the S-wave phase shift could be as large as $-7°$.

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I. INTRODUCTION

The CP-violating observable, A, in weak nonleptonic hyperon decays of the form $B\to B'\pi$ depends on the strong-rescattering phases of the final state [1]. At leading order, this asymmetry is given by

$$A=-\tan(\delta_S-\delta_P)\sin(\phi_S-\phi_P), \tag{1}$$

where δ_S and δ_P (ϕ_S and ϕ_P) are the strong-rescattering (weak) phases in the S- and P-wave components, respectively, of the decay amplitude. Currently the HyperCP (E871) experiment at Fermilab is in the process of measuring this CP-violating observable through the asymmetry sum $A(\Lambda)+A(\Xi)$ in the chain of decays $\Xi\to\Lambda\pi\to p\pi\pi$ [2]. Calculation of this observable, therefore, requires knowledge of both the phase shifts for $N\pi$ scattering at the Λ mass and those for $\Lambda\pi$ scattering at the Ξ mass. The former phase shifts have been extracted from experiment (albeit with large errors) [3], but there is no experimental data for the latter.

An early calculation [4] of the $\Lambda\pi$ scattering phase shifts at m_Ξ indicated that the S-wave phase shift was large, the result being $\delta_S=-18.7°$ and $\delta_P=-2.7°$. If correct, this would suggest that CP violation in both decays $\Xi\to\Lambda\pi$ and $\Lambda\to p\pi$ could yield similar contributions to the measurement of E871, making a theoretical prediction harder. More recently, this calculation has been repeated in the context of heavy-baryon chiral perturbation theory, with very different results. At leading order, it was found in Ref. [5] that $\delta_S=0$ and $\delta_P=-1.7°$. The implication of this result is that the CP-violating observable in E871 is probably dominated by CP violation in $\Lambda\to p\pi$. The vanishing of δ_S in this calculation results from the heavy-baryon limit. An estimate of relativistic corrections to the heavy-baryon result has been performed in Ref. [6], where it was found (within a leading-order calculation in the relativistic theory) that[1] $\delta_S=1.2°$ and $\delta_P=-1.7°$. Leading-order (in heavy-baryon chiral perturbation theory) calculations of the $N\pi$ [8] and $\Xi\pi$ [9] scattering phase shifts have also been carried out. They suggest that the smallness of δ_S at lowest order in chiral perturbation theory (χPT) is mostly a kinematic effect associated with the small pion-momentum available in $\Xi\to\Lambda\pi$.

Given the two very different results for δ_S in $\Lambda\pi$ scattering, it is important to estimate the effect of physics not present in the leading-order χPT calculation. A first attempt to investigate this question was carried out in Ref. [10]. Their approach was to look for possible resonant enhancements. To this effect, they considered the nearest resonance with the correct quantum numbers, the $\Sigma(1750)$ with $I=1, J^P=\frac{1}{2}^-$. Although the parameters of this resonance are not well known, the authors of Ref. [10] allowed them to vary in a reasonable range to conclude that the contribution to δ_S from this source was not more than about 0.5°.

In this paper we explore the possibility of an enhancement in δ_S due to the presence of nearby thresholds in other baryon-meson channels with the same quantum numbers. In particular, we wish to check the role of the S-wave $\Sigma\pi$ channel, which has a threshold only 10 MeV above m_Ξ. It is known from the Weinberg-Tomozawa theorem that the scattering length in this channel is very attractive [11]. To investigate this issue, we present two separate estimates. For the first one, we will take the point of view that any such effects can be parametrized by next-to-leading-order terms in chiral perturbation theory. The authors of Ref. [12] have recently studied the coupled-channel problem for the $S=-1, I=1$ baryon-meson system, within a certain model, and have parametrized their results in terms of specific values for the coupling constants in the heavy-baryon chiral Lagrangian up

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[1] We have redone this estimate and obtained the same result, but with δ_S having the opposite sign [7].

to the next-to-leading order $[\mathcal{O}(p^2)]$. We employ these values for the coupling constants in our first estimate. For our second estimate, we simply use leading-order χPT to derive all the amplitudes in the $S=-1,\ I=1$ baryon-meson system and employ a K-matrix formalism to incorporate the effects of unitarity in the coupled-channel problem.

II. $\mathcal{O}(P^2)$ HEAVY-BARYON CHIRAL LAGRANGIAN

We write the chiral Lagrangian for the strong interaction of the lightest (octet) baryons up to order p^2 in heavy-baryon χPT as the sum of two terms:

$$\mathcal{L}=\mathcal{L}^{(1)}+\mathcal{L}^{(2)}, \tag{2}$$

where the superscript refers to the chiral order. The first term is given by [13]

$$\mathcal{L}^{(1)}=\langle\bar{B}_v\,\mathrm{i}v\cdot\mathcal{D}B_v\rangle+2D\langle\bar{B}_vS_v^\mu\{\mathcal{A}_\mu,B_v\}\rangle+2F\langle\bar{B}_vS_v^\mu[\mathcal{A}_\mu,B_v]\rangle, \tag{3}$$

with $\langle\cdots\rangle\equiv\mathrm{Tr}(\cdots)$. The second term can be written in the most general form as [12,14]

$$\begin{aligned}\mathcal{L}^{(2)}=&\mathcal{L}_{\mathrm{rc}}^{(2)}+b_D\langle\bar{B}_v\{\chi_+,B_v\}\rangle+b_F\langle\bar{B}_v[\chi_+,B_v]\rangle+b_0\langle\chi_+\rangle\langle\bar{B}_vB_v\rangle+2d_D\langle\bar{B}_v\{(v\cdot\mathcal{A})^2,B_v\}\rangle+2d_F\langle\bar{B}_v[(v\cdot\mathcal{A})^2,B_v]\rangle\\&+2d_0\langle\bar{B}_vB_v\rangle\langle(v\cdot\mathcal{A})^2\rangle+2d_1\langle\bar{B}_vv\cdot\mathcal{A}\rangle\langle v\cdot\mathcal{A}B_v\rangle+2g_D\langle\bar{B}_v\{\boldsymbol{\mathcal{A}}\cdot\boldsymbol{\mathcal{A}},B_v\}\rangle+2g_F\langle\bar{B}_v[\boldsymbol{\mathcal{A}}\cdot\boldsymbol{\mathcal{A}},B_v]\rangle+2g_0\langle\bar{B}_vB_v\rangle\langle\boldsymbol{\mathcal{A}}\cdot\boldsymbol{\mathcal{A}}\rangle\\&+2g_1\langle\bar{B}_v\boldsymbol{\mathcal{A}}\rangle\cdot\langle\boldsymbol{\mathcal{A}}B_v\rangle+2h_D\langle\bar{B}_v\mathrm{i}\boldsymbol{\sigma}\cdot\{\boldsymbol{\mathcal{A}}\times\boldsymbol{\mathcal{A}},B_v\}\rangle+2h_F\langle\bar{B}_v\mathrm{i}\boldsymbol{\sigma}\cdot[\boldsymbol{\mathcal{A}}\times\boldsymbol{\mathcal{A}},B_v]\rangle+2h_1\langle\bar{B}_v\mathrm{i}\boldsymbol{\sigma}\times\boldsymbol{\mathcal{A}}\rangle\cdot\langle\boldsymbol{\mathcal{A}}B_v\rangle,\end{aligned} \tag{4}$$

where

$$\begin{aligned}\mathcal{L}_{\mathrm{rc}}^{(2)}=&\frac{-1}{2m_0}\langle\bar{B}_v[\mathcal{D}^2-(v\cdot\mathcal{D})^2]B_v-\bar{B}_v[S_v^\mu,S_v^\nu][[\mathcal{A}_\mu,\mathcal{A}_\nu],B_v]\rangle-\frac{\mathrm{i}D}{m_0}(\langle\bar{B}_vS_v\cdot\mathcal{D}\{v\cdot\mathcal{A},B_v\}\rangle+\langle\bar{B}_vS_v^\mu\{v\cdot\mathcal{A},\mathcal{D}_\mu B_v\}\rangle)\\&-\frac{\mathrm{i}F}{m_0}(\langle\bar{B}_vS_v\cdot\mathcal{D}[v\cdot\mathcal{A},B_v]\rangle+\langle\bar{B}_vS_v^\mu[v\cdot\mathcal{A},\mathcal{D}_\mu B_v]\rangle)-\frac{DF}{m_0}\langle\bar{B}_v[(v\cdot\mathcal{A})^2,B_v]\rangle-\frac{D^2}{2m_0}\langle\bar{B}_v\{v\cdot\mathcal{A},\{v\cdot\mathcal{A},B_v\}\}\rangle\\&-\frac{F^2}{2m_0}\langle\bar{B}_v[v\cdot\mathcal{A},[v\cdot\mathcal{A},B_v]]\rangle\end{aligned} \tag{5}$$

is the $1/m_0$ (leading relativistic) correction to the leading-order Lagrangian at order p^2, with m_0 being the octet-baryon mass in the chiral limit. The constants $b,\ d,\ g$ and h are free parameters (in addition to the familiar D and F) that occur at this order, and we will obtain their values from the model of Ref. [12]. In these formulas, B_v is the usual 3×3 matrix containing the (velocity dependent) octet-baryon fields, v the baryon velocity, S_v the spin operator, and

$$\mathcal{A}_\mu=\frac{\mathrm{i}}{2}(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi)=\frac{\partial_\mu\varphi}{2f}+\mathcal{O}(\varphi^3),$$

$$\begin{aligned}\chi_+&=\xi^\dagger M_\varphi^2\xi^\dagger+\xi M_\varphi^2\xi\\&=2M_\varphi^2-\frac{1}{4f^2}\{\varphi,\{\varphi,M_\varphi^2\}\}+\mathcal{O}(\varphi^4),\end{aligned} \tag{6}$$

where φ is the 3×3 matrix for the octet of pseudo-scalar bosons, $f=f_\pi=92.4$ MeV is the pion-decay constant, and $M_\varphi^2=\mathrm{diag}(m_\pi^2,m_\pi^2,2m_K^2-m_\pi^2)$ the pseudo-scalar mass matrix in the isospin limit.

The total amplitude for $\Lambda\pi\to\Lambda\pi$, up to order p^2, is derived from the diagrams in Fig. 1.[2] In the center-of-mass (c.m.) frame, it is given by

$$\begin{aligned}\mathcal{M}_{\Lambda\pi}=\frac{2m_\Lambda}{f^2}\chi_{\mathrm{f}}^\dagger\Bigg[&\frac{D^2}{3m_0}\left(m_\pi^2-\boldsymbol{k}^2+\frac{\boldsymbol{k}^4}{3E_\pi^2}\right)+\left(\frac{4b_D}{3}+4b_0\right)m_\pi^2\\&-\left(\frac{2d_D}{3}+2d_0\right)E_\pi^2+\frac{D^2}{3}\,\frac{3(\boldsymbol{k}'\cdot\boldsymbol{k})^2-\boldsymbol{k}^4}{3m_0E_\pi^2}\\&+\boldsymbol{k}'\cdot\boldsymbol{k}\left(\frac{2D^2}{3}\,\frac{m_\Sigma-m_\Lambda}{E_\pi^2}-\frac{2g_D}{3}-2g_0\right)\\&+\frac{2D^2}{3}\,\frac{\mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{k}'\times\boldsymbol{k}}{E_\pi}\left(1+\frac{E_\pi}{m_0}-\frac{\boldsymbol{k}^2+\boldsymbol{k}\cdot\boldsymbol{k}'}{2m_0E_\pi}\right)\Bigg]\chi_{\mathrm{i}},\end{aligned} \tag{7}$$

where χ_{i} (χ_{f}) is the Pauli spinor of the initial (final) Λ, and $\boldsymbol{k}$ ($\boldsymbol{k}'$) is the three-momentum of the initial (final) pion. The

[2] At this order there are no loop contributions. The latter begin at $\mathcal{O}(p^3)$.

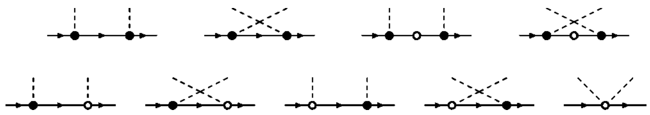

FIG. 1. Diagrams for $\Lambda\pi\to\Lambda\pi$ up to order p^2. A dashed (solid) line denotes a pion (octet baryon) field. The baryon in the intermediate states is Σ. Solid and open vertices are generated by $\mathcal{L}^{(1)}$ in Eq. (3) and $\mathcal{L}^{(2)}$ in Eq. (4), respectively.

partial-wave amplitudes are then extracted using standard techniques,[3] and one finds, in the $J=\frac{1}{2}$ channel,

$$f_{\Lambda\pi}^{(S)}=\frac{-m_\Lambda}{4\pi f^2\sqrt{s}}\left[\frac{D^2}{3m_0}\left(E_\pi^2-2\boldsymbol{k}^2+\frac{\boldsymbol{k}^4}{3E_\pi^2}\right)+\left(\frac{4b_D}{3}+4b_0\right)m_\pi^2 -\left(\frac{2d_D}{3}+2d_0\right)E_\pi^2\right], \tag{8}$$

$$f_{\Lambda\pi}^{(P)}=\frac{-\boldsymbol{k}^2 m_\Lambda}{4\pi f^2\sqrt{s}}\left[\frac{4D^2}{9E_\pi}\left(1+\frac{2E_\pi^2-\boldsymbol{k}^2}{2m_0E_\pi}+\frac{m_\Sigma-m_\Lambda}{2E_\pi}\right) -\frac{2}{9}(g_D+3g_0)\right]. \tag{9}$$

For our numerical calculation, we will adopt the parameter values provided by Ref. [12]. In that work, the chiral Lagrangian $\mathcal{L}$ in Eq. (2) was used as a starting point for constructing a coupled-channel potential model to study $N\bar{K}\to N\bar{K},\Lambda\pi,\Sigma\pi$ and other measured processes. We employ in particular the parameter values extracted in Ref. [16]:

$$\begin{aligned} &D=0.782, \quad m_0=0.869\ \text{GeV},\\ &b_0=-0.320\ \text{GeV}^{-1}, \quad b_D=0.066\ \text{GeV}^{-1},\\ &d_0=-0.996\ \text{GeV}^{-1}, \quad d_D=0.512\ \text{GeV}^{-1},\\ &g_0=-1.492\ \text{GeV}^{-1}, \quad g_D=0.320\ \text{GeV}^{-1}. \end{aligned} \tag{10}$$

Thus, with $\sqrt{s}=m_\Xi$ and $|\boldsymbol{k}|\simeq 0.137$ GeV, we obtain, for the $J=\frac{1}{2}$ channel, the phase shifts

$$\delta_S\simeq -2.5°, \quad \delta_P\simeq -3.3°. \tag{11}$$

Of course, the parameters in Eq. (10) are not known precisely. If we allow them to take the following ranges of values (in the same units as before),

$$\begin{aligned} &0.4<D<0.8, \quad 0.7<m_0<1.2,\\ &-0.6<b_0<-0.3, \quad 0.02<b_D<0.08,\\ &-1.0<d_0<-0.7, \quad 0.3<d_D<0.6,\\ &-1.5<g_0<-1.0, \quad 0.3<g_D<0.5, \end{aligned} \tag{12}$$

FIG. 2. Diagrams for $B\phi\to B'\phi'$ in the $J=\frac{1}{2}$ channel at leading order in χPT, including decuplet-baryon contributions. A dashed line denotes a meson field, and a single (double) solid-line denotes an octet-baryon (decuplet-baryon) field. Vertices are generated by $\mathcal{L}^{(1)}$ in Eq. (14).

which are suggested by various tree- and loop-level χPT calculations [13,17], as well as the results of Ref. [12], we find the following ranges for the phase shifts:

$$-3.0°<\delta_S<+0.4°, \quad -3.5°<\delta_P<-1.2°. \tag{13}$$

The contributions of the lowest-order terms to these numbers are $\delta_S^{(1)}=0$ and $-1.7°<\delta_P^{(1)}<-0.4°$. The $1/m_0$ terms, especially in the S wave, give small corrections, $-0.06°<\delta_S^{\rm rc}<-0.01°$ and $-0.4°<\delta_P^{\rm rc}<-0.1°$. Therefore, the rest of the p^2 terms in the S wave generate the bulk of δ_S, and those in the P wave are comparable to the lowest-order term in their contribution to δ_P.

It is useful to compare the result above with the leading-order result of Ref. [5]. In that calculation, the spin-$\frac{3}{2}$ decuplet-baryon degrees of freedom are also included in the chiral Lagrangian, so that, at leading order [13],

$$\begin{aligned}\mathcal{L}^{(1)}=&\langle\bar{B}_v \mathrm{i}v\cdot\mathcal{D}B_v\rangle+2D\langle\bar{B}_v S_v^\mu\{\mathcal{A}_\mu,B_v\}\rangle\\ &+2F\langle\bar{B}_v S_v^\mu[\mathcal{A}_\mu,B_v]\rangle-\bar{T}_v^\mu \mathrm{i}v\cdot\mathcal{D}T_{v\mu}+\Delta m\bar{T}_v^\mu T_{v\mu}\\ &+\mathcal{C}(\bar{T}_v^\mu\mathcal{A}_\mu B_v+\bar{B}_v\mathcal{A}_\mu T_v^\mu)+2\mathcal{H}\bar{T}_v^\mu S_v\cdot\mathcal{A}T_{v\mu},\end{aligned} \tag{14}$$

where T_v^μ represents the baryon-decuplet fields,[4] m_T is the decuplet-baryon mass in the chiral limit, and $\Delta m=m_T-m_0$. The resulting amplitude for $\Lambda\pi\to\Lambda\pi$ in the $J=\frac{1}{2}$ channel receives nonzero contributions from the last three diagrams in Fig. 2 and is given by

$$\begin{aligned}\mathcal{M}_{\Lambda\pi}=&\frac{2m_\Lambda}{f^2}\chi_{\rm f}^\dagger\boldsymbol{k}\cdot\boldsymbol{k}'\left(\frac{\frac{1}{3}D^2}{\sqrt{s}-m_\Sigma}-\frac{\frac{1}{3}D^2}{E_\pi-E_\Lambda+m_\Sigma}\right.\\ &\left.-\frac{\frac{1}{6}\mathcal{C}^2}{E_\pi-E_\Lambda+m_{\Sigma^*}}\right)\chi_{\rm i}+\frac{2m_\Lambda}{f^2}\chi_{\rm f}^\dagger \mathrm{i}\boldsymbol{\sigma}\cdot\boldsymbol{k}'\times\boldsymbol{k}\left(\frac{\frac{1}{3}D^2}{\sqrt{s}-m_\Sigma}\right.\\ &\left.+\frac{\frac{1}{3}D^2}{E_\pi-E_\Lambda+m_\Sigma}-\frac{\frac{1}{12}\mathcal{C}^2}{E_\pi-E_\Lambda+m_{\Sigma^*}}\right)\chi_{\rm i},\end{aligned} \tag{15}$$

the Σ and Σ^* being the intermediate baryons. This result, at order p^1, actually contains some contributions from the chiral Lagrangian of order p^2 which are implicit in the denominators. The partial-wave amplitudes in the $J=\frac{1}{2}$ channel are then

[3] See, e.g., Ref. [15].

[4] For hadronic fields, we follow the notation of Ref. [18].

$$f_{\Lambda\pi}^{(S)}=0,\quad f_{\Lambda\pi}^{(P)}=\frac{-\boldsymbol{k}^2 m_\Lambda}{4\pi f^2\sqrt{s}}\left(\frac{\frac{1}{3}D^2}{\sqrt{s}-m_\Sigma}+\frac{\frac{1}{9}D^2}{E_\pi-E_\Lambda+m_\Sigma}-\frac{\frac{1}{9}\mathcal{C}^2}{E_\pi-E_\Lambda+m_{\Sigma^*}}\right).\tag{16}$$

Consequently, using $\sqrt{s}=m_\Xi$ and the tree-level values[5] $D=0.8$ and $\mathcal{C}=1.7$, we find for the $J=\frac{1}{2}$ channel the phase shifts

$$\delta_S=0,\quad \delta_P\simeq-1.5°.\tag{17}$$

If chiral-symmetric masses $m_\Sigma=m_0\simeq 1.15$ GeV and $m_{\Sigma^*}=m_T\simeq 1.38$ GeV are used instead, we obtain $\delta_P\simeq-1.0°$. In each of these δ_P values, roughly $-2°$ arises from the two diagrams involving the Σ, and about $+0.8°$ comes from the diagram containing the Σ^*. One can see that the value of the Σ contribution is compatible with the $\delta_P^{(1)}$ range quoted above. However, the Σ^* contribution is opposite in sign to all the $\mathcal{O}(p^2)$ contributions to δ_P that we have estimated using the Lagrangian in Eq. (4). This suggests that there are additional contributions beyond that of the Σ^* which can be expected to be significant.

In Ref. [6], the baryons are not treated as heavy and the phase shifts are computed using the relativistic version of the lowest-order Lagrangian in Eq. (14). We have repeated the calculation (with $D=0.8$ and $\mathcal{C}=1.7$) and found $\delta_S=-1.2°$ and $\delta_P=-1.7°$, which agrees with the result of Ref. [6], except that δ_S has the opposite sign.[6] In δ_S here, only $-0.1°$ is generated by the Σ-mediated diagrams, with the rest, $-1.1°$, coming from the Σ^*-mediated diagram. We can see that the Σ contribution is comparable to the $\delta_S^{\rm rc}$ values in the $\mathcal{O}(p^2)$ heavy-baryon estimate. In contrast, the dominant Σ^* contribution is roughly only half of the δ_S value in the $\mathcal{O}(p^2)$ heavy-baryon result, although the two have the same sign. This again suggests that other contributions in addition to that of the Σ^* may be important. In the P wave, the Σ and Σ^* contributions to δ_P are $-2.7°$ and $+1.0°$, respectively, and so these are similar to their $\mathcal{O}(p^1)$ heavy-baryon counterparts.

III. *K*-MATRIX APPROACH

The SU(3) picture that we have implies that the $\Lambda\pi$ state is coupled to the states $\Sigma\pi$, $N\bar{K}$, $\Sigma\eta$, and ΞK with $S=-1$ and $I=1$. Thus the $\Lambda\pi$ scattering can be treated as a problem with five coupled channels. Although at $\sqrt{s}=m_\Xi$ all the inelastic channels are below threshold, they may significantly affect the elastic one through unitarity constraints. The inclusion of such kinematically closed channels has been recently shown to be important in the case of $N\bar{K}$ interactions [19].

In order to estimate the impact on the $\Lambda\pi$ channel of the others coupled to it, we employ a K-matrix approach. This method guarantees that the resulting partial-wave amplitudes satisfy unitarity exactly. We follow the formalism described in Ref. [15]. For the K-matrix elements, we will make the simplest approximation and use only the partial-wave amplitudes at leading order in χPT, obtained from $\mathcal{L}^{(1)}$ in Eq. (14).

The relevant partial waves can be extracted by choosing the five isospin states

$$\begin{aligned}
&|\Lambda\pi,I=1\rangle=|\Lambda\pi^-\rangle,\\
&|\Sigma\pi,I=1\rangle=\frac{1}{\sqrt{2}}(|\Sigma^0\pi^-\rangle-|\Sigma^-\pi^0\rangle),\\
&|N\bar{K},I=1\rangle=-|nK^-\rangle,\\
&|\Sigma\eta,I=1\rangle=|\Sigma^-\eta\rangle,\\
&|\Xi K,I=1\rangle=-|\Xi^-K^0\rangle.
\end{aligned}\tag{18}$$

The phase convention here is consistent with the structure of the φ and B_v matrices.

The lowest-order S-wave amplitude for $B\phi\to B'\phi'$ with $S=-1$ and $I=1$ is derived from the first diagram in Fig. 2 and, in the c.m. frame, has the form

$$f_{B\phi,B'\phi'}^{(S)}=-C_{B\phi,B'\phi'}\sqrt{m_B m_{B'}}\frac{E_\phi+E_{\phi'}}{16\pi f_\pi^2\sqrt{s}},\tag{19}$$

where $C_{B'\phi',B\phi}=C_{B\phi,B'\phi'}$. Using the isospin states above, one obtains

$$\begin{aligned}
&C_{\Lambda\pi,\Lambda\pi}=C_{\Lambda\pi,\Sigma\pi}=0,\quad C_{\Lambda\pi,N\bar{K}}=\sqrt{\frac{3}{2}},\quad C_{\Lambda\pi,\Sigma\eta}=0,\\
&C_{\Lambda\pi,\Xi K}=\sqrt{\frac{3}{2}},\quad C_{\Sigma\pi,\Sigma\pi}=-2,\quad C_{\Sigma\pi,N\bar{K}}=-1,\\
&C_{\Sigma\pi,\Sigma\eta}=0,\quad C_{\Sigma\pi,\Xi K}=+1,\\
&C_{N\bar{K},N\bar{K}}=-1,\quad C_{N\bar{K},\Sigma\eta}=\sqrt{\frac{3}{2}},\quad C_{N\bar{K},\Xi K}=0,\\
&C_{\Sigma\eta,\Sigma\eta}=0,\quad C_{\Sigma\eta,\Xi K}=\sqrt{\frac{3}{2}},\quad C_{\Xi K,\Xi K}=-1.
\end{aligned}\tag{20}$$

The resulting K matrix is written as

$$K=\begin{pmatrix}K_{\rm oo} & K_{\rm oc}\\ K_{\rm co} & K_{\rm cc}\end{pmatrix},\tag{21}$$

where, with $f_{B\phi,B'\phi'}\equiv f_{B\phi,B'\phi'}^{(S)}$,

[5]These are extracted from hyperon semileptonic decays (which also give $F=0.5$) and the strong decays $T\to B\phi$, respectively.

[6]We have checked that expanding the part of the amplitude arising from the Σ diagrams up to order p^2 does lead to the D^2 terms in the heavy-baryon $\mathcal{O}(p^2)$ amplitude in Eq. (7).

$$K_{\rm oo}=f_{\Lambda\pi,\Lambda\pi}\,, \quad K_{\rm co}=K_{\rm oc}^{\rm T}=\begin{pmatrix} f_{\Lambda\pi,\Sigma\pi} \\ f_{\Lambda\pi,N\bar{K}} \\ f_{\Lambda\pi,\Sigma\eta} \\ f_{\Lambda\pi,\Xi K}\end{pmatrix},$$

$$K_{\rm cc}=\begin{pmatrix} f_{\Sigma\pi,\Sigma\pi} & f_{\Sigma\pi,N\bar{K}} & f_{\Sigma\pi,\Sigma\eta} & f_{\Sigma\pi,\Xi K} \\ & f_{N\bar{K},N\bar{K}} & f_{N\bar{K},\Sigma\eta} & f_{N\bar{K},\Xi K} \\ & & f_{\Sigma\eta,\Sigma\eta} & f_{\Sigma\eta,\Xi K} \\ & & & f_{\Xi K,\Xi K}\end{pmatrix} = K_{\rm cc}^{\rm T}, \tag{22}$$

the subscripts ''o'' and ''c'' referring, respectively, to open and closed channels at $\sqrt{s}=m_\Xi$. The unitarized S-wave amplitude for $\Lambda\pi\to\Lambda\pi$ is then given by [15]

$$T_{\rm oo}=\frac{K_{\rm r}}{1-{\rm i}q_{\rm o}K_{\rm r}}=\frac{{\rm e}^{2{\rm i}\delta_S}-1}{2{\rm i}|\boldsymbol{k}_{\Lambda\pi}|}, \tag{23}$$

where

$$K_{\rm r}=K_{\rm oo}+{\rm i}K_{\rm oc}(1-{\rm i}q_{\rm c}K_{\rm cc})^{-1}q_{\rm c}K_{\rm co}\,,$$

$$q_{\rm o}=q_{\Lambda\pi}\,, \quad q_{\rm c}={\rm diag}(q_{\Sigma\pi},q_{N\bar{K}},q_{\Sigma\eta},q_{\Xi K}), \tag{24}$$

with $q_{B\phi}=|\boldsymbol{k}_{B\phi}|$, the magnitude of the particle three-momentum in $B\phi\to B\phi$ scattering in the c.m. frame. We note that $T_{\rm oo}$ not only satisfies elastic unitarity exactly, but also reproduces the lowest-order χPT amplitude $f_{\Lambda\pi,\Lambda\pi}$ (which happens to vanish in the S-wave case) as the chiral limit is approached. We further note that the diagonal elements of $q_{\rm c}$ are purely imaginary at $\sqrt{s}=m_\Xi$, their corresponding channels being below threshold. It follows that the S-wave phase shift in $\Lambda\pi\to\Lambda\pi$ scattering at $\sqrt{s}=m_\Xi$ is calculated to be

$$\delta_S=\tan^{-1}(q_{\Lambda\pi}K_{\rm r})\simeq-7.3°. \tag{25}$$

If we drop the heavier $\Sigma\eta$ and ΞK channels, the phase shift is reduced in size to $\delta_S\simeq-2.8°$. If only the ΞK channel is dropped, we find instead $\delta_S\simeq-3.6°$. These numbers are consistent with the fact that the $\Lambda\pi$ state has nonzero S-wave couplings at leading order only to the $N\bar{K}$ and ΞK states. Interestingly, the last two numbers, $\delta_S\sim-3°$, are similar to the δ_S in Eq. (11), calculated using χPT at order p^2 with the parameter values from Ref. [12], in which the heavier channels were not explicitly considered. In Fig. 3 we show the real part of δ_S (which becomes complex above the $\Sigma\pi$ threshold) as a function of the center-of-mass energy, with all the four inelastic channels contributing.

We remark here that we have not included contributions from the $B\phi\phi'$ states in our K matrix as they are not expected to be dominant, only entering at the two-loop level in χPT. Similarly, with the lowest-order vertices that we have, there are no S-wave couplings to states with a decuplet baryon and a pseudo-scalar meson.

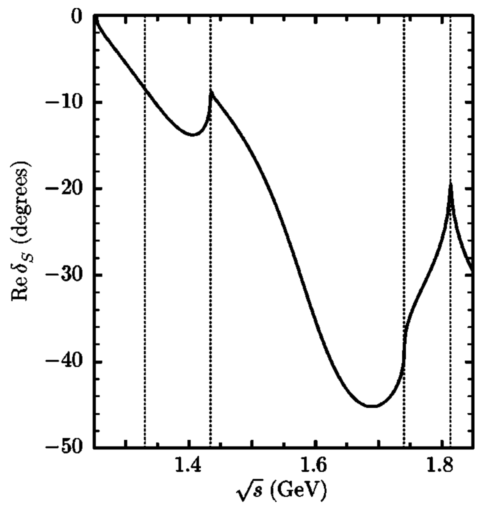

FIG. 3. S-wave phase shift in $\Lambda\pi$ scattering as a function of the center-of-mass energy. The dotted lines mark the thresholds of the $\Sigma\pi$, $N\bar{K}$, $\Sigma\eta$, and ΞK channels, respectively. (Note that $m_\Xi\simeq1.32$ GeV is just 10 MeV below the $\Sigma\pi$ threshold.)

The leading-order P-wave amplitude for $B\phi\to B'\phi'$ with $S=-1$ and $I=1$ is derived from the last three diagrams in Fig. 2. In the c.m. frame, its $J=\frac{1}{2}$ component can be written in the form

$$f^{(P)}_{B\phi,B'\phi'}=-D_{B\phi,B'\phi'}\frac{|\boldsymbol{k}_\phi||\boldsymbol{k}_{\phi'}|\sqrt{m_B m_{B'}}}{8\pi f^2\sqrt{s}}, \tag{26}$$

where $D_{B'\phi',B\phi}=D_{B\phi,B'\phi'}$, and $\boldsymbol{k}_\phi$ ($\boldsymbol{k}_{\phi'}$) is the three-momentum of the initial (final) meson. We have collected in the Appendix the expressions for $D_{B\phi,B'\phi'}$ corresponding to the five coupled channels. The P-wave $J=\frac{1}{2}$ phase shift in $\Lambda\pi\to\Lambda\pi$ scattering at $\sqrt{s}=m_\Xi$ is then calculated using the same method as in the S-wave case. The result for $D=0.8$, $F=0.5$, and $\mathcal{C}=1.7$ is

$$\delta_P\simeq0.2°, \tag{27}$$

where isospin-symmetric masses have been used for the intermediate baryons. If chiral-symmetric masses $m_0\simeq1.15$ GeV and $m_T\simeq1.38$ GeV are used for the intermediate baryons, the result is instead

$$\delta_P\simeq0.5°. \tag{28}$$

We have found that dropping one or more of the inelastic channels would not change these numbers dramatically in size, yielding a phase shift within the range $-1.5°<\delta_P<-0.4°$.

As in the S-wave case, we have not included contributions from the $B\phi\phi'$ states in our P-wave K matrix. Neither have we considered states with only one decuplet baryon and no meson in the s channel as they have $J=\frac{3}{2}$, but they were included as intermediate states in the $J=\frac{1}{2}$ u channel. Beyond the simplest approximation that we have made, one could add contributions from heavier states, such as those with one decuplet baryon and one pseudo-scalar meson, as well as those with heavier resonances. The neglect of heavier states is taken as part of the uncertainty of our estimate.

IV. CONCLUSION

We have studied the $\Lambda\pi$ scattering phase shifts at $\sqrt{s}=m_\Xi$ beyond leading order in chiral perturbation theory. With next-to-leading-order χPT, we find results that are consistent within factors of 2 with the lowest-order phase shifts. Within a *K*-matrix approach, we find that unitarity effects from coupled channels enhance the *S*-wave phase shift, which could be as large as $-7°$, but they do not change the *P*-wave phase shift significantly. The large *S*-wave value is driven mostly by couplings to the heavier channels. Since the two approaches do not incorporate exactly the same physics, their results may be combined to indicate that $-7.3°<\delta_S<+0.4°$ and $-3.5°<\delta_P<+0.5°$, leading to $-7.8°<\delta_S-\delta_P<+3.9°$. Our results also indicate that more-refined future calculations with χPT as a starting point should include the effects of coupled channels, as has been done in the $N\bar{K}$ case [12,19,20]. Finally, it is possible to extract these phase shifts from experiment. We expect E871 to present results in the near future from their analysis of polarization in $\Xi\to\Lambda\pi$ decays.

Note added. After this paper was submitted for publication, Ref. [21] appeared. Using dispersion relations in a relativistic chiral unitary approach based on coupled channels, its authors obtained $0\leqslant\delta_S\leqslant1.1°$ and claimed that our *K*-matrix result should not be trusted.

All of the existing perturbative (χPT) calculations consistently predict a small δ_S. The question of whether this phase shift can be large is motivated by the implications that this would have for experiments searching for *CP* violation. The ultimate and definitive resolution of this question lies in experiment, and we hope that E871 can shed some light on this issue in the near future. At present, speculation on a possible large contribution to δ_S involves models beyond chiral perturbation theory, so that it is very difficult to estimate an uncertainty. We have presented a *K*-matrix result which suggests that δ_S could be as large as $-7°$, but we cannot estimate reliably the uncertainty in this number. The authors of Ref. [21] agree that our calculation is technically correct, and so their conclusion that our large result should not be trusted is only a statement of their preference for their model over a simple *K* matrix. They justify this preference from fitting, in Ref. [20], to the data in the $N\bar{K}$, $\Sigma\pi$, and $\Lambda\pi$ channels. However, a simple look at some of their results (Fig. 2 of Ref. [20]) indicates that not all of the data are well reproduced by their model, and this suggests to us that the resulting δ_S could have a large uncertainty. Furthermore, in our *K*-matrix calculation the largest contribution to δ_S arises from the ΞK channel, and we currently have no way of knowing whether the model of Ref. [21] reproduces the scattering amplitudes for this channel. Finally, in Ref. [21] two values for δ_S are presented, corresponding to two different parameter sets in their model. It is amazing that whereas the two sets lead to minimal differences in the description of the scattering amplitudes of Ref. [20], their predictions for δ_S differ by a factor of almost 10. This further suggests that there could be a rather large uncertainty in these predictions.

We are led to conclude that until we have an experimental result for δ_S as guidance, it is not really possible to prefer one model over another. What is interesting at this stage is that there is at least one model (the simple *K* matrix) which suggests that the phase shift can be larger than the predictions of chiral perturbation theory.

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APPENDIX

For the five coupled channels considered here, one obtains

$$D_{\Lambda\pi,\Lambda\pi}=\frac{\frac{2}{3}D^2}{\sqrt{s}-m_\Sigma}-\frac{\frac{2}{9}D^2}{E_\Lambda-E_\pi-m_\Sigma}+\frac{\frac{2}{9}\,\mathcal{C}^2}{E_\Lambda-E_\pi-m_{\Sigma*}},\tag{A1}$$

$$D_{\Lambda\pi,\Sigma\pi}=\frac{\frac{4}{\sqrt{6}}DF}{\sqrt{s}-m_\Sigma}+\frac{\frac{4}{3\sqrt{6}}DF}{E_\Lambda-E'_\pi-m_\Sigma}+\frac{\frac{4}{9\sqrt{6}}\mathcal{C}^2}{E_\Lambda-E'_\pi-m_{\Sigma*}},$$

$$D_{\Lambda\pi,N\bar{K}}=\frac{-\sqrt{\frac{2}{3}}D(D-F)}{\sqrt{s}-m_\Sigma}-\frac{\frac{1}{3\sqrt{6}}(D^2+4DF+3F^2)}{E_\Lambda-E'_K-m_N},$$

$$D_{\Lambda\pi,\Sigma\eta}=\frac{\frac{2}{3}D^2}{\sqrt{s}-m_\Sigma}+\frac{\frac{2}{9}D^2}{E_\Lambda-E'_\eta-m_\Lambda},$$

$$D_{\Lambda\pi,\Xi K}=\frac{-\sqrt{\frac{2}{3}}D(D+F)}{\sqrt{s}-m_\Sigma}-\frac{\frac{1}{3\sqrt{6}}(D^2-4DF+3F^2)}{E_\Lambda-E'_K-m_\Xi}-\frac{\frac{4}{9\sqrt{6}}\mathcal{C}^2}{E_\Lambda-E'_K-m_{\Xi*}},$$

$$D_{\Sigma\pi,\Sigma\pi}=\frac{4F^2}{\sqrt{s}-m_\Sigma}+\frac{\frac{2}{9}D^2}{E_\Sigma-E'_\pi-m_\Lambda}-\frac{\frac{2}{3}F^2}{E_\Sigma-E'_\pi-m_\Sigma}+\frac{\frac{2}{27}\mathcal{C}^2}{E_\Sigma-E'_\pi-m_{\Sigma*}},$$

$$D_{\Sigma\pi,N\bar{K}}=\frac{-2(D-F)F}{\sqrt{s}-m_\Sigma}+\frac{\frac{1}{3}(D^2-F^2)}{E_\Sigma-E'_K-m_N}-\frac{\frac{8}{27}\mathcal{C}^2}{E_\Sigma-E'_K-m_\Delta},$$

$$D_{\Sigma\pi,\Sigma\eta}=\frac{\frac{4}{\sqrt{6}}DF}{\sqrt{s}-m_\Sigma}-\frac{\frac{4}{3\sqrt{6}}DF}{E_\Sigma-E'_\eta-m_\Sigma}+\frac{\frac{4}{9\sqrt{6}}\mathcal{C}^2}{E_\Sigma-E'_\eta-m_{\Sigma*}},$$

$$D_{\Sigma\pi,\Xi K}=\frac{-2(D+F)F}{\sqrt{s}-m_\Sigma}-\frac{\frac{1}{3}(D^2-F^2)}{E_\Sigma-E'_K-m_\Xi}-\frac{\frac{4}{27}\mathcal{C}^2}{E_\Sigma-E'_K-m_{\Xi*}}, \tag{A2}$$

$$D_{N\bar{K},N\bar{K}}=\frac{(D-F)^2}{\sqrt{s}-m_\Sigma}, \tag{A3}$$

$$D_{N\bar{K},\Sigma\eta}=\frac{-\frac{\sqrt{6}}{3}D(D-F)}{\sqrt{s}-m_\Sigma}-\frac{\frac{\sqrt{6}}{18}(D^2-4DF+3F^2)}{E_N-E'_\eta-m_N},$$

$$D_{N\bar{K},\Xi K}=\frac{D^2-F^2}{\sqrt{s}-m_\Sigma}-\frac{\frac{1}{18}(D^2-9F^2)}{E_N-E'_K-m_\Lambda}+\frac{\frac{1}{6}(D^2-F^2)}{E_N-E'_K-m_\Sigma}+\frac{\frac{2}{27}\mathcal{C}^2}{E_N-E'_K-m_{\Sigma*}},$$

$$D_{\Sigma\eta,\Sigma\eta}=\frac{\frac{2}{3}D^2}{\sqrt{s}-m_\Sigma}-\frac{\frac{2}{9}D^2}{E_\Sigma-E'_\eta-m_\Sigma}+\frac{\frac{2}{9}\mathcal{C}^2}{E_\Sigma-E'_\eta-m_{\Sigma*}},$$

$$D_{\Sigma\eta,\Xi K}=\frac{-\frac{\sqrt{6}}{3}D(D+F)}{\sqrt{s}-m_\Sigma}-\frac{\frac{\sqrt{6}}{18}(D^2+4DF+3F^2)}{E_\Sigma-E'_K-m_\Xi}-\frac{\frac{2\sqrt{6}}{27}\mathcal{C}^2}{E_\Sigma-E'_K-m_{\Xi*}}, \tag{A4}$$

$$D_{\Xi K,\Xi K}=\frac{D^2+2DF+F^2}{\sqrt{s}-m_\Sigma}+\frac{\frac{4}{9}\mathcal{C}^2}{E_\Xi-E'_K-m_\Omega}, \tag{A5}$$

where $\sqrt{s}=E_B+E_\phi$ and E'_ϕ is the energy of ϕ in the final state.

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