Essays on Self-fulfilling Expectations and Business Cycles

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THESIS

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Abstract

This thesis studies the self-fulfilling business cycles in a dynamic stochastic general equilibrium model with financial market frictions. It consists of three papers.

The first paper uncovers a series of belief shocks (a.k.a animal spirits) that drive the U.S. economy from both financial markets data and the structure of a financial accelerator model with borrowing constraint. It finds that the computed belief shocks are well identified and resemble the observable proxy in the real world. Furthermore, the model economy in which only sunspot shocks matter performs at least as well as a standard real business cycle model driven by technology shocks in replicating major U.S. business cycle facts and it outperforms the real business cycle model in some other dimensions.

The second paper investigates the role of people's animal spirits in an estimated artificial economy with financial market frictions via Bayesian methods. It demonstrates that people's animal spirits are prime drivers of U.S. business cycle fluctuations. Animal spirits shocks account for well over a third of output fluctuations over the period from 1955 to 2014. Financial friction and technology shocks are considerably less important. It also finds that a substantial part of aggregate output's contraction during the Great Recession was caused by adverse shocks to expectations.

The third paper follows the path of Adelman and Adelman (1959), applying the classical business cycle method proposed by Burns and Mitchell (1946) to evaluate the cyclical properties of an animal spirits model that is estimated in the second paper. In particular, the paper examines whether the model can reproduce qualitative features of U.S. business cycle. The results indicate an adequately high degree of coincidence in main macroeconomic aggregates between the business cycle features identified in actual time series data and those found in model economy.

Declaration

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and, to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. I addition, I certify that no part of this work will, in the future, be used in a submission in my name for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and where applicable, any partner institution responsible for the joint-award of this degree.

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Chapter 1

Introduction

"The indeterminacy of equilibrium is not a problem to be avoided by clever assumptions; it is a fact that can be exploit to explain the world." [Farmer, 1997, 605]

This thesis empirically explores the role of self-fulfilling expectations in U.S. aggregate fluctuations and aims at contributing to the understanding of the Keynesian idea of fluctuations driven by animal spirits. The following three papers provide empirical evidence in support of belief-driven business cycle fluctuations.

"Animal spirits", "self-fulfilling expectations" or "sunspots" represent unexplained waves of optimism and pessimism of agents which may give rise to economic instability. If the animal spirits are high, confidence will be boosted among participants of the economy then economic growth will be stimulated. Similarly, if the spirits are low, people are in bad faith then a promising market will be driven down even though the economic fundamentals stay strong. An example is the 1990-91 recession in which consumer and business confidence depreciated as a result of oil price shock, coupled with an already weak economy. The 2007-08 financial crisis triggered the renewed interest in the role that animal spirits play in the financial markets. From over-credulity in ever-increasing housing prices to plunging confidence in capital markets, animal spirits are driving financial events worldwide.

The theoretical framework of this thesis is a dynamic stochastic general equilibrium model incorporating both multiple equilibria and financial market frictions. Particularly, the analysis is implemented by building on a variant of Benhabib and Wang (2013) in which the interaction between collateral constraint and endogenous markup give rise to equilibrium indeterminacy, a condition that allows aggregate fluctuations to be caused by extrinsic changes to people's expectations. There are two main reasons that motivate the choice of such a theoretical cornerstone: (i) The early endogenous business cycle models involve strong economy-wide increasing returns to scale for sunspot equilibria to exist that are not suggested by empirical studies such as Basu and Fernald (1997). Moreover, Lubik (2016) suggests that the model with other mechanisms than production externality seems a more promising candidature in studying sunspot business cycles. (ii) The global financial crisis of 2007-2008 forced macroeconomists to rethink about their analytical frameworks. Hence models featuring financial frictions are promptly entering the mainstream of macro modeling. Benhabib and Wang (2013) is a notable sample of the recent studies in the field. The remainder of the introduction provides a more detailed description of each core paper.

The first paper uncovers a sequence of belief shocks that impinges on the economy and examines the empirical validity of sunspot-driven business cycle models in some aspects. Specifically, following Sayler and Sheffrin (1998) who conceive of the idea that belief shocks that drive the economic fluctuations also drive asset return, the paper identifies belief shocks from a discrete time version of Benhabib and Wang's financial accelerator model that features self-fulfilling prophecies. I use actual time series to compute the implied values of the belief shock and the sample period is quarterly spanning the period from 1967: I to 2015: IV. The paper finds that the computed belief shocks and technology shocks are meaningfully labeled and have the resemblance to observable proxies in the real world. I also feed the estimated shocks into the model and compare the efficacy of the sunspot-driven model with the standard RBC model. The results show that: (i) Self-fulfilling beliefs can generate dynamics that resemble the empirical ordering of cyclical volatilities and contemporaneous correlations which are comparable to predictions of standard RBC model. (ii) The artificial model driven by i.i.d belief shocks can generate hump-shaped impulse response pattern that traditional RBC models fail to explain. (iii) By running a Fair and Shiller (1990) test, the model with self-fulfilling expectations outperforms the standard RBC model. (iv) The counterfactual exercise implies that the model driven by self-fulfilling beliefs matches the U.S. time series data reasonably well which contrasts with the standard RBC model driven by technology shocks.

The second paper asks: what are the shocks that cause observed business cycle fluctuations? The paper attends to this question by presenting evidence on sources of business cycles for the post-Korean War American economy. The undertaking is implemented by building on a variant of Benhabib and Wang (2013). We modify the original model by incorporating a parade of fundamental shocks which are frequently considered as key drivers of business cycles. The model is estimated by full information Bayesian methods using quarterly U.S. data covering the period from 1955:I to 2014:IV. The estimation results support the view that people's animal spirits play a significant role in the U.S. business cycle. In particular, variance decomposition suggest that animal spirits are be-

hind around forty percent of output growth variations and they explain an even larger portion of fluctuations in investment spending. Technology shocks and financial shocks are significantly less important and they explain no more than twenty percent of the oscillations in aggregate real economic activity. Besides, our counterfactual exercise reveals that the Great Recession, for the most part, was caused by adverse shocks to expectations. Finally, we compare the empirical fit of the model with determinacy versus indeterminacy. We find that the indeterminate model in which animal spirits play a significant role turns out to be empirically superior.

The third paper conducts Adelman tests for a self-fulfilling business cycle model using Burns and Mitchell approach. Burns and Mitchell investigate the central characteristics of the business cycle of time series by constructing the reference-cycle pattern which is a necessary tool to examine the cyclical behavior of different economic time series. The common strategy for assessing a model is to compare the second-moment behavior of artificial series with their actual counterparts. The classical method of Burns and Mitchell, instead, provides both visual evaluations (e.g. nine-point plot) and descriptive statistics (e.g. amplitudes and conformity) of business cycle properties. I use a series of belief shocks estimated from the second chapter as model driving forces and evaluate the cyclical properties of this animal spirits model. Especially, the paper examines if one could distinguish between the actual historical time series and the artificial series generated by a stochastically perturbed economic model. The results indicate that the artificial series generated by animal spirits model shows a high degree of coincidence with the data series not only for the general shape but also for the volatility and coherence.

Chapter 2

Endogenous Business Cycles with Financial Frictions

2.1 Introduction

Since the pathbreaking work of Benhabib and Farmer (1994), there is a rapidly growing interest in studying expectations-driven macroeconomic fluctuations. The main reason for this is that it provides the possibility of quantitative analysis of sunspot equilibria within the framework of Kydland and Prescott (1982). The existence of a continuum of equilibria in Benhabib-Farmer model relies on two mechanisms, one with increasing returns to scale at the aggregate level via external effects and the other one with increasing returns to scale at the firm level by way of monopolistic competition. The postulated degrees of economies of scale that are required to obtain indeterminacy, however, has been widely criticized as being unrealistic (see Aiyagari, 1995; Basu and Fernald, 1997). Since then, a line of research has been devoted to bringing the degree of returns to scale for indeterminacy down to an empirically plausible range.¹ A more recent generation of RBC models shows that production externalities are not at all needed for indeterminacy to arise plausibly (for example, Bennett and Farmer, 2000; Nakajima, 2006; Jaimovich, 2007; Krause and Lubik, 2010; Benhabib and Wang, 2013; Liu and Wang, 2014). The current paper stands in line with these studies.

One implication of indeterminacy for the theory of business cycle is that the existence of a continuum of equilibria offers an alternative source of impulses to business fluctuations – disturbances to expectations (a.k.a. beliefs or animal spirits). As the seminal research of Farmer and Guo (1994) showed, the economy may display fluctuations at the business cycle frequencies even in the absence of any underlying fundamental uncertainty. Measuring belief shocks is problematic since they are unobservable much like preference. One possible way is to use Monte Carlo methods to generate belief shocks and then compute model statistics (e.g. Farmer and Guo, 1994). An alternative approach is to use actual time series data and additional model information to pin down the belief shock once the belief function of agents has been specified (e.g. Salyer and Sheffrin, 1998). Chauvet and Guo (2003) and Harrison and Weder (2006) try to extract idiosyncratic sunspot shocks by constructing a vector autoregression (VAR) model in which the residual from a regression of the empirical proxy for confidence on fundamental serve as belief shocks.

This paper pursues the second approach to identify an observable counterpart to the non-fundamental shock that impinges on the economy. In particular, I follow Salyer and Sheffrin (1998) who conceive of the idea that belief shocks that drive the economic fluctuations also drive asset return. While the method

¹Examples include, among others, Benhabib and Farmer (1996), Wen (1998), Weder (2000) and Guo and Harrison (2001).

of computing beliefs remains the same, the underlying theoretical model of this study is different from theirs. They uncover belief shocks from a version of the Farmer and Guo model while I identify belief shocks from a discrete time version of Benhabib and Wang's (2013) financial accelerator model that features self-fulfilling prophecies. The reason is twofold. First, the Farmer-Guo economy requires unrealistically large increasing returns to scale while indeterminacy can arise in the Benhabib-Wang model with constant social returns. Second, the general equilibrium models with financial constraints initiated by Kiyotaki and Moore (1997) and others have been widely used after the recent financial crisis of 2007-2008. I use actual data to compute the implied values of the belief shock to study the role of beliefs empirically in the U.S. economy. The sample period is quarterly over the period 1967:I-2015:IV. The computed belief shocks are well identified and resemble the observable proxy in the real world.

I feed the estimated shocks into the model and assess the sunspots-driven fluctuations. I also compare the efficacy of the self-fulfilling expectations model with the standard real business cycle model in several dimensions. The paper finds that: (a) The animal spirits model is able to replicate the regular features of U.S. business cycles and is comfortably comparable to the standard RBC model under technology shocks. (b) The model is driven by i.i.d expectational shocks contains a strong endogenous propagation mechanism, and it can generate hump-shaped impulse response pattern of the U.S. business cycle that traditional RBC models driven by technology shocks fail to explain. (c) By running a Fair and Shiller (1990) test, the model with self-fulfilling beliefs outperforms the standard RBC model. (d) The counterfactuals provide some empirical support for the role of non-fundamental rational expectations in economic fluctuations. Artificial business cycles of belief-driven model match the U.S. time series data well which is in sharp contrast to that of RBC paradigm.

The remainder of the paper proceeds as follows. Section 2.2 outlines the model. Belief shocks are computed and compared with empirical proxies in Section 2.3. Section 2.4 makes a comparison between self-fulfilling expectation model and standard real business cycle model in various dimensions. Section 2.5 concludes.

2.2 The Model

The artificial economy is a discrete-time version of Benhabib and Wang (2013). The model framework closely follows the original paper; hence, expository comments will be brief. The critical feature of their model is that intermediate good producing firms are collateral constrained in how much they can borrow to finance their working capitals. The mechanism generating a self-fulfilling explanation stands on an endogenous and countercyclical markup channel.

2.2.1 Firms

The economy consists of two sectors. Perfectly competitive firms produce final output y_t combining a continuum of intermediate inputs $y_t(i)$ according to the technology

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\lambda-1}{\lambda}} di\right)^{\frac{\lambda}{\lambda-1}},\tag{2.1}$$

where $\lambda \geq 1$ controls the elasticity of substitution between input varieties. Each monopolistic competitors *i* produce intermediate goods and has access to a constant returns technology given by,

$$y_t(i) = z_t k_t(i)^{\alpha} h_t(i)^{1-\alpha},$$
 (2.2)

where $0 < \alpha < 1$. All firms are equally affected by aggregate total factor productivity, z_t , whose law of motion is given by

$$\ln z_t = (1 - \rho) \ln z + \rho \ln z_{t-1} + w_t \qquad 0 < \rho < 1, \tag{2.3}$$

where $w_t \sim N(0, \sigma_w^2)$ is the innovation term driving this process. Unlike final good producers, these intermediate firms are credit constrained for working capital needs. Imperfect enforcement requires a process to constrain borrowing by the value of the collateral. Specifically, the total amount of debt (i.e. an intraperiod loan) $b_t(i)$ is constrained by the value of the collateral which assume is the end-of-period of assets, i.e.

$$b_t(i) = w_t h_t(i) + r_t k_t(i) \le \xi p_t(i) y_t(i) - f.$$
(2.4)

Under this credit constraint, if there is a default event, the household has the right to recover a fraction $\xi < 1$ of firm's output being produced by incurring a liquidation cost f. Denoting marginal cost $\phi_t = 1/A_t(r_t/\alpha)^{\alpha}(w_t/(1-\alpha))^{1-\alpha}$, the financial constraint becomes

$$\phi_t y_t(i) \le \xi p_t(i) y_t(i) - f. \tag{2.5}$$

2.2.2 Households

The representative household chooses paths for consumption c_t and total hours of work h_t so as to maximize its lifetime utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) - \varphi \frac{h_t^{(1+\eta)}}{1+\eta} \right], \qquad (2.6)$$

subject to the period-by-period budget constraint

$$k_{t+1} = (1 - \delta_t)k_t + w_t h_t + r_t u_t k_t - c_t + \pi_t, \qquad (2.7)$$

$$\delta_t = \delta_0 \frac{u_t^{1+\nu}}{1+\nu},\tag{2.8}$$

where $0 < \beta < 1$ is the subjective discount factor, $\eta \ge 0$ measures the inverse Frisch elasticity of substitution for labour supply and φ measures the disutility for working. The rate of capital depreciation, δ_t , is an increasing and convex function of capacity utilization. $\delta_0 \in (0, 1)$ is a constant and $\nu > 0$ measures the elasticity of the depreciation rate with respect to capacity utilization. With higher capital utilization, capital depreciates faster.

2.2.3 Equilibrium and Dynamics

In symmetric equilibrium, $k_t(i) = u_t k_t$, $h_t(i) = h_t$, $p_t(i) = p_t$, $y_t(i) = y_t$, $\pi_t(i) = \pi_t = y_t - w_t h_t - r_t u_t k_t$ and the collateral constraint binds. A competitive equilibrium in this model is characterized by the following necessary conditions:

$$\frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} \left(1 - \delta_{t+1} + \phi_{t+1} \frac{\alpha y_{t+1}}{k_{t+1}} \right) \right], \tag{2.9}$$

$$k_{t+1} = y_t + (1 - \delta_t)k_t - c_t, \qquad (2.10)$$

$$\varphi h_t^{1+\eta} = \phi_t \frac{(1-\alpha)y_t}{c_t},\tag{2.11}$$

$$y_t = z_t (u_t k_t)^{\alpha} h_t^{(1-\alpha)},$$
 (2.12)

$$\delta_0 u_t^{1+\nu} = \phi_t \frac{\alpha y_t}{k_t},\tag{2.13}$$

$$\phi_t = \xi - \frac{f}{y_t}.\tag{2.14}$$

The first equation is the intertemporal Euler condition. The second expression depicts the economy-wide resource constraint as reflected in the law of motion for the aggregate capital stock. Equation (2.11) describes the consumptionleisure trade-off. Equation (2.12) is the production function. Equation (2.13) determines the efficient level of capacity utilization and the last equation is the binding collateral constraint. Notice that the model is reduced to a standard RBC model if f = 0 (i.e., ϕ_t is constant).

I take log-linear approximation to the equilibrium conditions to obtain the following dynamic system:

$$\begin{bmatrix} \hat{c}_{t+1} \\ \hat{y}_{t+1} \\ \hat{z}_{t+1} \end{bmatrix} = J \begin{bmatrix} \hat{c}_t \\ \hat{y}_t \\ \hat{z}_t \end{bmatrix} + M \begin{bmatrix} \hat{\varepsilon}_{t+1} \\ \hat{w}_{t+1} \end{bmatrix}, \qquad (2.15)$$

where hat variables denote percent deviations from their steady-state values. The local dynamics are determined by the roots of Jocobian Matrix J. Here $\hat{\varepsilon}_{t+1}$ is the belief shock which is assumed independently and identically distributed over time. Indeterminacy of rational expectations requires that both eigenvalues of J are inside the unit circle. When indeterminacy arises, equilibria may be driven by belief shocks. Since all variables have now been log-linearized, the remaining endogenous variables can be expressed as a linear combination of consumption, output and TFP:

$$\begin{bmatrix} \hat{k}_t \\ \hat{n}_t \\ \hat{h}_t \\ \hat{u}_t \\ \hat{\phi}_t \end{bmatrix} = Q \begin{bmatrix} \hat{c}_t \\ \hat{y}_t \\ \hat{z}_t \end{bmatrix}, \qquad (2.16)$$

where Q is a matrix determined by the log-linearization.

2.3 Computing belief shocks

2.3.1 Belief shocks

I firstly generate the technology shock as residuals from a Solow decomposition. Measurement takes place within the structure of a production function, adjusting for capital utilization.² The total factor productivity is given by

$$z_t = \frac{y_t}{(u_t k_t)^{\alpha} h_t^{1-\alpha}},$$
(2.17)

where y_t , k_t , h_t and u_t are real per capita GDP, capital stock, labour supply and capacity utilization.³ The log-linearly detrend series, \hat{z}_t , is used as the technology shock which is well described by a first order autoregressive process with $\rho = 0.96$.

Once the technology shocks are pinned down, the model structure allows the identification of belief noise. Since financial market is believed to be driven in

²The calibration uses the parameter values in Benhabib and Wang (2013): $\beta = 0.99$, $\alpha = 1/3$, $\eta = 0$, $\lambda = 10$, $\delta = 0.0333$, $\nu = 0.3$, $\phi = 0.88$, $\xi = 0.9768$, f = 0.1908.

 $^{^{3}}$ The capital stock was constructed by the perpetual inventory mehod, taking into account the variable depreciation rate. See Appendix for the source and construction of U.S. data.

part by expectations, using realized asset returns may help us to better understand and isolate the source of the belief shocks.

Assuming that shares held during period (t-1), s_{t-1} , yield a dividend payment d_t at time t; time-t equity prices are q_t . Households maximize expected lifetime utility by financing consumption c_t from an exogenous stochastic dividend stream, proceeds from sales of share, and an exogenous stochastic endowment m_t . The optimization is subject to

$$c_t + q_t(s_t - s_{t-1}) = d_t s_{t-1} + m_t.$$
(2.18)

The necessary condition is the familiar intertemporal condition

$$q_t \frac{1}{c_t} = \beta E_t \left[\frac{1}{c_{t+1}} (q_{t+1} + d_{t+1}) \right].$$
(2.19)

Setting R_t as the gross return on equity, equation (2.19) can be written as

$$\frac{1}{c_t} = E_t \left[\frac{\beta}{c_{t+1}} (R_{t+1}) \right].$$
 (2.20)

Then, denoting linearized variables with hats, equation (2.20) becomes

$$c_t = E_t \hat{c}_{t+1} - E_t \hat{R}_{t+1}.$$
(2.21)

The equilibrium function for consumption in the calibrated model is

$$\hat{c}_{t+1} = 0.89\hat{c}_t + 0.013\hat{y}_t + 0.21\hat{z}_t + \hat{\varepsilon}_{t+1}, \qquad (2.22)$$

Since $\hat{\varepsilon}_{t+1} = \hat{c}_{t+1} - E_t \hat{c}_{t+1}$, we can get

$$E_t \hat{c}_{t+1} = 0.89 \hat{c}_t + 0.013 \hat{y}_t + 0.21 \hat{z}_t. \tag{2.23}$$

The linearized structure of the economy implies that realized equity returns will be a linear function of last period's state variables and the current belief and technology shocks. Setting

$$\hat{R}_{t+1} = \alpha_0 \hat{c}_t + \alpha_1 \hat{y}_t + \alpha_2 \hat{z}_t + \alpha_3 \hat{\varepsilon}_{t+1} + \alpha_4 \hat{z}_{t+1}, \qquad (2.24)$$

then

$$E_t \hat{R}_{t+1} = \alpha_0 \hat{c}_t + \alpha_1 \hat{y}_t + (\alpha_2 + \rho \alpha_4) \hat{z}_t.$$
(2.25)

Substituting equation (2.23) and equation (2.25) into equation (2.21), one can solve $\alpha_0 = -0.11$, $\alpha_1 = 0.013$, and $\alpha_2 + \rho \alpha_4 = 0.21$. As a result, the realized returns on equity implies that the belief shock can be observed by calculating

$$(\alpha_3 - 1)\hat{\varepsilon}_{t+1} = \hat{c}_t + \hat{R}_{t+1} - \hat{c}_{t+1} - \alpha_4 \hat{w}_{t+1}.$$
(2.26)

I assume that the coefficient on the belief shock in equation (2.26) is equal to unity. The value of α_4 is chosen so that the belief shock and the innovation to technology are orthogonal. Using U.S. data on gross return on equity, consumption and technology innovation, one can get the value of α_4 by regressing $\hat{\theta}_{t+1} = \hat{c}_t + \hat{R}_{t+1} - \hat{c}_{t+1}$ on \hat{w}_{t+1} and uncover the belief shock. Over the sample period 1967:I to 2015:IV, this regression implies $\alpha_4 = 1.78$.⁴

Figure 2.1 shows the estimated belief shocks over the sample along with the

⁴Since the model variables are defined as percentage deviation from the steady state, I use the deviation from a common linear trend estimated for all series except for gross return on equity. Equity returns are expressed as deviations from the mean.



Figure 2.1: Belief shock as identified from artificial economy

NBER recession dates. As can be seen from the figure, the process appears to be the white noise, which is required for the rational expectation models. The recessions in the U.S. economy usually correspond to decrease in beliefs. Besides, I also employ the Breusch-Godfrey test to check the serial correlation. As evidenced in Table 2.1, the sequence of implied belief shocks is in line with the white noise assumption. For serial correlation up to fourth-order, the Breusch-Godfrey test statistics do not reject the null of no serial correlation.

Lags	F-statistic	p-values
4	1.76	0.14

Table 2.1: Breusch-Godfrey serial correlation LM test

2.3.2 Shocks and empirical proxies

In the preceding analysis, a series of animal spirits shocks and technology shocks are identified from the structure of the model. I now compare the calculated shocks with the observable proxies in the real world to check whether



Figure 2.2: Consumer confidence index vs Belief shocks

they are meaningfully identified. In particular, the implied belief shock and total factor productivity are compared with the University of Michigan's Consumer Confidence Index and Fernald's (2014) utilization-adjusted TFP measure, respectively. All series are bandpass-filtered to capture business cycle frequencies only. Figures 2.2 and 2.3 shows that all theoretical shocks are positively correlated with their empirical counterparts. Furthermore, belief shocks and confidence typically drop during recessions.

2.4 Empirical results and model comparison

In this section, the calculated shocks are fed into the linearized version of the theoretical model. I investigate and compare the dynamic properties of macroeconomic time series generated by the self-fulfilling expectation (SFE) model and real business cycle (RBC) model in four dimensions.



Figure 2.3: Fernald's TFP vs Model's TFP

2.4.1 Second moments

I first examine whether the model-generated time series fit post-war U.S. business cycles fluctuations by investigating the contemporaneous moments. The predicted second moments of business cycles as implied by the model and as measured in U.S. data are reported in Table 2.2. The main stylized facts of business cycles can be observed from the U.S. sample moments: (1) consumption, investment and labour hours are positively correlated with aggregate output; (2) the detrended components in these aggregate quantities are all highly persistent; (3) consumption fluctuates less than output, investment displays more volatile than output, and labour hours are roughly as volatile as output.

The performance of the self-fulfilling expectation model relative to a standard RBC model with variable capital utilization in explaining the cyclical properties of U.S. data can be summarized as follows. In both models, the variance of the shocks can be chosen so that the predicted variance of simulated output matches the actual data exactly. Therefore, the persistence is the only thing

Relative volatilities	σ_y/σ_y	σ_c/σ_y	σ_i/σ_y	σ_h/σ_y
U.S. economy	1.00	0.39	2.50	0.85
SFE model	1.00	0.06	4.32	1.08
RBC model	1.00	0.22	3.78	0.81
Correlations	$cor(y_{t},y_{t})$	$cor(c_{t},y_{t})$	$cor(i_t, y_t)$	$cor(h_{t}, y_{t})$
U.S. economy	1.00	0.85	0.98	0.88
SFE model	1.00	0.53	0.99	0.99
RBC model	1.00	0.87	0.99	0.99
Autocorrelations	$cor(y_{t}, y_{t-1})$	$cor(c_{t}, c_{t-1})$	$cor(i_{t,i_{t-1}})$	$cor(h_{t}, h_{t-1})$
U.S. economy	0.94	0.92	0.93	0.95
SFE model	0.94	0.96	0.94	0.94
RBC model	0.91	0.93	0.91	0.91

Table 2.2: U.S. sample and model moments

Notes: U.S. sample period: 1967:I - 2015:IV. (y, c, i, h) stand for output, consumption, investment and labour hours. Both U.S. sample and model series are detrended by bandpass filter.

that matters to output and both models predict the autocorrelation fairly well. With respect to consumption, the RBC model can explain the relative volatilities close to the data whereas the indeterminate model substantially underpredicts it. With respect to investment, both models can explain its excess volatility relative to output. With respect to labour hours volatility, the two models basically can match the data reasonably well while the SFE model is a slightly over-predict the relative volatility. Besides, both models are quite successful in matching the positive comovement between consumption, investment, hours and output as well as the serial correlations. In particular, the animal spirits or SFE model possesses a strong internal propagation mechanism which generates highly autocorrelated variable. In opposition to existing RBC models, this persistence arises without the help of highly autocorrelated forcing variables.

Overall, therefore, in terms of the conventional measures of the business cycle, it is fair to say that the model in which i.i.d belief shocks are the sole disturbances performs equally as well as real business cycle paradigm. The distinct features of data can be replicated.

2.4.2 Impulse response functions

To check another dimension of the dynamic properties of the two models, I look at the impulse response functions that trace out how a dynamical system reacts to an initial shock. Figure 2.4 shows impulse responses of the model's key variables to a one-time positive belief shock (solid lines) as well as to an innovation of one standard deviation to the technology shock (dashed lines) for the first 40 quarters.



Figure 2.4: Impulse responses of output, consumption, investment and hours. Solid lines are responses to a belief shock, dashed lines are responses to a technology shock.

By comparison, some features of Figure 2.4 deserve particular attention. First, an increase in both belief shocks and technology shocks can induce positive comovement among output, consumption, investment and hours. Second, the model with self-fulfilling beliefs generates endogenous, highly persistent, and hump-shaped dynamics. Third, without indeterminacy, the impulse response of macroeconomic variables to technology shocks monotonically declines with time. The monotonic response in the RBC model implies that it will not be able to generate cyclical and autocorrelated behavior similar to that seen in the actual data unless the technology shock itself is highly autocorrelated. Such failure has been criticized by the researcher over a long time.

The impulse for the self-fulfilling economy is a spontaneous change in agents' expectations. The optimistic representative household expects higher future output and thus the higher value of collateral. Regarding the collateral constraints equation (2.14), the increase in the value of the collateral, other things being equal, eases the borrowing constraint raising the firms' borrowing ability so that the unit marginal cost can rise and markup can fall. The factor payments will be pushed up with output due to the fierce competition for labour and capital among firms. The labour demand curve is upward sloping and steeper than that of labour supply curve due to the counter-cyclical markup channel in this economy. A rise in output creates a positive wealth effect, which increases the demand for both consumption and leisure. The labour supply curve shifts inwards with the income effect, thereby raising employment robustly. The increase in hours, as well as the accumulation of capital, raise the level of output, allowing the initial beliefs about higher output to become self-fulfilling. This sequence of impact events is quite different from the origins of the cycle within a real business cycle economy, where a positive technology shock shifts the marginal product of labour schedule outward, causing higher equilibrium employment, output, consumption, and investment.

2.4.3 Pictures of the data

For expectational shocks to be an acceptable explanation for business cycles, however, it is vital that the implications be supported by empirical evidence. In this part, I present the simulated times series of SEF model and RBC model and compare them with the actual U.S. time series. I adjust the standard deviation of shocks in a way that causes volatility of output is the same for the model and the data over the whole sample. Figures 2.5 and 2.6 present the responses of output, consumption, investment and hours worked in these two model economies for a single simulation experiment, and I plot the series for each model alongside the actual U.S. time series. All series have been passed through the bandpass filter.



Figure 2.5: The SFE economy



Figure 2.6: The RBC economy

As can be seen from the Figure 2.5, the SFE model tracks all the series reasonably well except that the predicted volatility of consumption by the model is too smooth relative to the data. The smoothness of consumption in the model is expected, given the inclusion of variable capacity utilization in the model. Variable capital utilization allow for an extra margin of adjustment, making risk-averse agents extremely smooth consumption (see also Wen, 1998 and Benhabib and Wen, 2004). Although the match is far from perfect, it is encouraging. In particular, the analysis shows that self-fulfilling pessimism may have played a nontrivial role in the 1973-1975 recession, the 2001 recession, and the 2007-2008 global financial crisis. The RBC model driven by the estimate technology shocks, however, do not match the U.S. time series data well because the estimated Solow residual tend to be uncorrelated with the aggregate output (see Burnside et al, 1996). These figures provide some empirical support for the role of nonfundamental rational expectations in economic fluctuations.

2.4.4 Forecasting exercise

In this part, I apply a test that aims to assess the forecasting ability and additional power that each model provides. I employ the spirit of Fair and Shiller (1990) and compare the information content implied by the endogenous variables of the model through the lens of a regression. For instance, the SFE and RBC model's forecasts for model variables can be compared via the regression:

$$xus_t = \alpha + \beta_1 xsfe_t + \beta_2 xrbc_t + \epsilon_t, \qquad (2.27)$$

where $x_t = (y, c, i, h)$, us denotes US data, sfe represents SFE model, and rbc means RBC mode. Throughout, all variables are deviations from the steadystate which is defined as the trend predicted by the bandpass filter. The null hypothesis is that neither model provides information, $\beta_1 = \beta_2 = 0$; if the SFE model provides information while the RBC model does not, the null is $\beta_1 \neq 0$, $\beta_2 = 0$; and vice versa.

	Output	Consumption	Investment	Hours
β_1 (SFE model)	0.37*	2.97*	0.23*	0.14*
	(0.00)	(0.00)	(0.00)	(0.01)
β_2 (RBC model)	-0.10	0.28*	-0.10*	-0.52*
	(0.18)	(0.02)	(0.03)	(0.00)

Table 2.3: Forecast comparison of RBC and SFE models

Notes: A * indicates significance at the 5 percent level (P-values are in parentheses).

From Table 2.3, it can be noticed that the SFE model provides statistically significant information for the behavior of all series. Specifically, SFE model contributes positively and significantly to U.S. output. The RBC model, however, provides no statistically significant additional help in explaining output above and beyond the SFE model. In terms of consumption, the coefficient is positive and significant in both models. With respect to investment and labour hours, SFE model provides additional positive explanation power while RBC model contributes negatively. In a nutshell, the results from this exercise imply that the model with self-fulfilling beliefs dominates the RBC model disturbed by technology shocks.

2.5 Conclusion

This paper uncovers a series of non-fundamental shock that impinges on the economy. The result of the paper indicates that the belief shocks are well identified and resemble the observable proxy in the real world. Furthermore, the model that beliefs are the only source of fluctuation is quite successful in replicating major U.S. business cycle facts. This implies that the possibility that cycles may be driven, at least in part, by self-fulfilling beliefs is plausible. In contrast to a standard Real Business Cycle models, the animal spirits model contains a strong internal propagation mechanism and has more forecasting ability. The results provide some empirical support for the class of macroeconomic model with multiple equilibria whereby agent's self-fulfilling beliefs exert an important role in economic fluctuations.
2.A Appendix

2.A.1 Data Sources

This appendix provides detailed information about the source and construction of the data used in the paper. All data are quarterly and are in real, per capita terms for the sample period 1967:I-2015:IV.

 Personal Consumption Expenditures, Nondurable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

 Personal Consumption Expenditures, Services. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Durable Goods. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

4. Gross Private Domestic Investment. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Domestic Product. Seasonally adjusted at annual rates, billions of dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

 Gross Domestic Product. Seasonally adjusted at annual rates, billions of chained (2009) dollars. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

 Nonfarm Business Hours. Index 2009=100, seasonally adjusted. Source: Bureau of Labour Statistics, Series Id: PRS85006033.

8. Civilian Noninstitutional Population. 16 years and over, thousands. Source: Bureau of Labour Statistics, Series Id: LNU00000000Q. 9. GDP Deflator = (5)/(6).

10. Real Per Capita Consumption, $c_t = [(1) + (2)]/(9)/(8)$.

- 11. Real Per Capita Investment, $i_t = [(3) + (4)]/(9)/(8)$.
- 12. Real Per Capita Output, $y_t = (10) + (11)$.
- 13. Per Capita Hours Worked, $h_t = (7)/(8)$.

14. Capacity Utilization: Total Industry (TCU), Percent of Capacity, Seasonally Adjusted, Source: Board of Governors of the Federal Reserve System.

15. Stock Market Data (S&P 500), retrieved from Shiller's website

(http://www.econ.yale.edu/~shiller/data.htm).

	Animal Spirits, Financial Markets, and Aggregate Instability						
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Chapter 3

Animal Spirits, Financial Markets and Aggregate Instability

3.1 Introduction

What are the shocks that cause macroeconomies to experience recurrent sequences of booms and slumps? The current paper pursues this question by presenting evidence on the sources of business cycles for the post-Korean War American economy. The results support the view that people's psychological motivations, a.k.a. animal spirits, provoke a significant portion of the fluctuations in aggregate real economic activity, causing well over one third of U.S. output volatility. This finding is demonstrated within an artificial economy of financial market frictions. Our exercise also suggests that it was chiefly adverse shocks to expectations that led to the Great Recession. Models with credit market frictions have become popular since the Great Recession, reflecting the notion that disruptions to financial markets were the key factors behind this contraction. Building on earlier work, such as Kiyotaki and Moore (1997) as well as Bernanke et al. (1999), this research has shown how financial market frictions can amplify shocks to macroeconomic fundamentals by transforming small economic disturbances into large business cycles.¹ Christiano et al. (2015), for example, extend New Keynesian models by financial market frictions to explain some key aspects of the Great Recession.

We depart from the aforementioned works twofold. First, the parametric space of our model includes multiple equilibria. This multiplicity will be cleared up by people's animal spirits that select from the possible equilibrium outcomes. Second, unlike most existing work on such indeterminacy, the analysis concentrates on estimating the artificial economy: we focus on the empirical implications of the multiplicity by explicitly analyzing the business cycle variance contributions of animal spirits or belief shocks. The undertaking is implemented by building on a variant of Benhabib and Wang (2013).² Indeterminacy in this model is linked to the empirically observed countercyclical movement of financial market tightness. Figure 3.1 plots the cyclical pattern of financial market health. It measures financial health by the Baa Corporate Bond spread which is displayed on an inverted scale and is plotted opposite the fluctuations of per capita GDP. The shaded areas in the figure correspond to NBER recessions. They highlight that financial conditions are not only cyclical, but also deteriorate markedly during most slumps.

In the artificial economy, countercyclical financial health is a key mechanism

¹See also Liu et al. (2013) and Nolan and Thoenissen (2009).

²Azariadis et al. (2016), Liu and Wang (2014) and Harrison and Weder (2013) are other models of various stripes that combine multiple equilibria and financial frictions.



Figure 3.1: U.S. GDP and credit spread (on right-hand scale) at business cycle frequencies. Shaded areas indicate NBER recessions.

to multiplicity. It is the endogenous interaction of a time varying (flow) collateral constraint and a countercyclical markup that spawns equilibrium indeterminacy, a condition that allows aggregate fluctuations to be caused by extrinsic changes in people's expectations. Moreover, in addition to such animal spirits shocks, the economy is buffeted by an array of fundamental shocks. The model is estimated by full information Bayesian methods using quarterly U.S. data covering the period from 1955:I to 2014:IV. This approach follows for example Justiniano et al. (2011) as well as Schmitt-Grohé and Uribe (2012), who, however, only explore the role of fundamental shocks as the engines of business cycles. The key result that ensues from the Bayesian estimation is that animal spirits are important drivers of the repeated fluctuations of the U.S. macroeconomy. Specifically, by computing forecast error variance decompositions, we find that animal spirits account for about 40 percent of U.S. output variations and for about two thirds of the fluctuations in investment. Disturbances that originate in the financial sector explain less than ten percent of output fluctuations. Moreover, we show that belief shocks have played an important role in the sharp contraction in economic activity of the Great Recession that began at the end of 2007.

Previous work on multiple equilibria in real economies has overwhelmingly remained in the theoretical realm and estimation exercises have been rare. Farmer and Guo (1995) is an early attempt to estimate a sunspot model using classical simultaneous equations methods. It is only Pintus et al. (2016) and Pavlov and Weder (2017) who perform full-information Bayesian estimations as in the present paper. Pintus et al. (2016) build a model with financial market frictions and loan contracts that are arranged with variable-rates of interest. The model's indeterminacy affects the propagation mechanism in particular of (fundamental) financial shocks. These shocks then explain about one quarter of business cycles fluctuations. Financial markets are not featured in Pavlov and Weder (2017) and their study excludes the Great Recession. Lastly,while the exact definitions of confidence do not completely overlap, our result also parallels Angeletos et al. (2016) and Milani (2017) who maintain that sentiment swings drive a large fraction of U.S. aggregate fluctuations.

Next, we will lay out the artificial economy. This is followed by the presentation of the estimation, discussions of results and various robustness checks. Finally, we provide a theory of the Great Recession.

3.2 The Model

The artificial economy features credit frictions in the form of endogenous borrowing constraints in a model of monopolistic competition in which, as usual, perfectly competitive firms produce final output by combining a continuum of differentiated intermediate inputs. Intermediate goods producing firms are collateral-constrained in how much they can borrow to finance their working capital needs. We modify the original model by incorporating a set of fundamental shocks which are frequently considered as key drivers of business cycles. Time proceeds in discrete steps. The model's discussion will be relatively brief and it will concentrate on the alterations to Benhabib and Wang (2013).

3.2.1 Technology

A unit mass of monopolistic competitive firms has access to a constant returns technology that transforms capital services $\kappa_t(i)$ and labor hours $N_t(i)$ into intermediate, differentiated outputs $Y_t(i)$

$$Y_t(i) = \kappa_t(i)^{\alpha} (X_t N_t(i))^{1-\alpha} \qquad 0 < \alpha < 1.$$

Exogenous labor-augmenting technological progress X_t affects all firms equally. Its growth rate $\mu_t^x \equiv X_t/X_{t-1}$ evolves as a first-order autoregressive process

$$\ln \mu_t^x = (1 - \rho_x) \ln \mu^x + \rho_x \ln \mu_{t-1}^x + \varepsilon_{x,t} \qquad 0 < \rho_x < 1$$

with $\varepsilon_{x,t} \sim N(0, \sigma_x^2)$ and $\ln \mu^x$ is average growth rate. The firms rent the two factor services from the households at perfectly competitive prices w_t and r_t . Final output Y_t is a constant elasticity of substitution aggregator of a basket of intermediate inputs

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\lambda-1}{\lambda}} di\right)^{\frac{\lambda}{\lambda-1}} \qquad \lambda > 1.$$

Here λ denotes the elasticity of substitution between the differentiated varieties. The monopolistic competitive firms generate profits by charging a mark-up over marginal costs. Following Barth and Ramey (2001) who report that a substantial portion of U.S. firms raise working capital, we assume that firms' two variable inputs must be financed by short-run loans. Imperfect enforcement requires a process to constrain borrowing by the value of the collateral. Specifically, firm *i*'s total amount of debt is an intraperiod loan $B_t(i)$ and it is constrained by the value of the collateral, which is the firms' pledge of the period-earnings, i.e.

$$B_t(i) = w_t N_t(i) + r_t \kappa_t(i) \le \theta_t \xi_t P_t(i) Y_t(i).$$

Under this credit constraint, if there is a default event, the lender has the right to recover a fraction of the firm's end-of-period revenues $P_t(i)Y_t(i)$.³ The model features two financial frictions and their product $\theta_t \xi_t$ represents the artificial economy's financial tightness. Concretely, ξ_t refers to an endogenous credit constraint: the borrowing constrictions vary with the aggregate state of economic activity which reflects creditors' ability to pay back loans. In particular, ξ_t is an increasing function of the deviation of actual output Y_t from balanced-growth output \bar{Y}_t

$$\xi_t = \tau \left(\frac{Y_t}{\bar{Y}_t}\right)^{\gamma}$$

in which we restrict the parameter to $0 < \tau < 1$ and $\gamma > 0$, an assumption in line with Figure 3.1. The parsimonious formulation of ξ_t entails many micro-founded makeups without the need to confine itself to a particular one.⁴ For example, it can stand in for Benhabib and Wang's (2013) setup with fixed liquidation costs or ξ_t can also describe how market conditions determine the probability that lenders can recover as well as resell collateral. In addition to the endogenous component,

 $^{^{3}}$ Unlike in the original Benhabib and Wang (2013) model, our setup does not include fixed liquidation costs. Indeterminacy still holds. When we compare the two models using the Bayesian estimation method, we find that the model without fixed costs is favored by the data.

⁴Eisfeldt and Rampini (2006) offer some evidence about the cyclical properties of ξ_t .

exogenous disturbances θ_t affect financial health. These shocks originate in the financial sector as in Jermann and Quadrini (2012) or Liu et al. (2013). The exogenous collateral or financial shock θ_t evolves as

$$\ln \theta_t = (1 - \rho_\theta) \ln \theta + \rho_\theta \ln \theta_{t-1} + \varepsilon_{\theta,t} \qquad 0 < \rho_\theta < 1$$

with $\varepsilon_{\theta,t} \sim N(0, \sigma_{\theta}^2)$ and steady state value $\theta = 1$. The corresponding first-order conditions for the profit maximization problem involve

$$r_t \kappa_t(i) = \alpha \phi_t Y_t(i)$$

$$w_t N_t(i) = (1 - \alpha)\phi_t Y_t(i)$$

and

$$\frac{\lambda - 1}{\lambda} P_t(i) - \phi_t + \mu_t(i) \left[\theta_t \xi_t \frac{\lambda - 1}{\lambda} P_t(i) - \phi_t \right] = 0$$
(3.1)

where ϕ_t stands for monopolistic firms' marginal costs and $\mu_t(i)$ denotes the multiplier associated with the borrowing constraint.

3.2.2 Preferences

Households are represented by an agent with the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t - \Lambda_t) - \varphi \frac{N_t^{1+\eta}}{1+\eta} \right) \qquad 0 < \beta < 1, \eta \ge 0 \text{ and } \varphi > 0$$

where β is the discount factor, C_t stands for consumption, and N_t for total hours worked. The functional form of the period utility ensures that the economy is consistent with balanced growth. The parameter φ denotes the disutility of working. The term Λ_t represents perturbations to the agent's utility of consumption that generate urges to consume, as in Baxter and King (1991) and Weder (2006). This element comes in two parts. One part grows with economy's consumption trend and the other one is a transitory shock that follows the autoregressive process

$$\ln \Delta_t = \rho_\Delta \ln \Delta_{t-1} + \varepsilon_{\Delta,t} \qquad 0 < \rho_\Delta < 1$$

with $\varepsilon_{\Delta,t} \sim N(0, \sigma_{\Delta}^2)$. This shock is also one of the drivers of the economy's labor wedge, i.e. the gap between the marginal rate of consumption-leisure substitution and the marginal product of labor. Hence, our estimation will allow a wider interpretation than mere shocks to preferences. A more agnostic reading includes, for example, wage or price stickiness, changes to monetary policy, taxes, or labor market frictions. Households own the physical capital stock K_t and decide on its utilization rate, u_t , thus $\kappa_t = u_t K_t$. The agent faces the period budget constraint

$$C_t + A_t I_t + T_t = w_t N_t + r_t u_t K_t + \Pi_t$$

and the law of motion for capital is

$$K_{t+1} = (1 - \delta_t)K_t + I_t.$$

The term I_t is investment spending and A_t represents a non-stationary investmentspecific technology shock which affects the transformation of consumption goods into investment goods. In the model, the concept corresponds to the relative price of new investment goods in terms of consumption goods. The shock's growth rate μ^a_t evolves as

$$\ln \mu_t^a = (1 - \rho_a) \ln \mu^a + \rho_a \ln \mu_{t-1}^a + \varepsilon_{a,t} \qquad 0 < \rho_a < 1$$

with $\varepsilon_{a,t} \sim N(0, \sigma_a^2)$, and $\ln \mu^a$ is the average growth rate. Lump-sum taxes are denoted by T_t . The rate of physical capital depreciation

$$\delta_t = \delta_0 \frac{u_t^{1+\nu}}{1+\nu}$$
 $0 < \delta_0 < 1$ and $\nu > 0$

is an increasing function in the utilization and $\nu > 0$ measures the elasticity of the depreciation rate with respect to capacity used. The first-order conditions are standard and delegated to the Appendix.

3.2.3 Government

The government purchases G_t units of the final output. G_t is neither productive nor does it provide any utility. The spending is financed by the lump-sum taxes. We model government's spending with a stochastic trend

$$X_t^G = (X_{t-1}^G)^{\psi_{yg}} (X_{t-1}^Y)^{1-\psi_{yg}} \qquad 0 < \psi_{yg} < 1$$

where ψ_{yg} governs the smoothness of the government spending trend relative to the trend in output. Then, detrended government spending is $g_t \equiv G_t/X_t^G$ and this follows the process

$$\ln g_t = (1 - \rho_g) \ln g + \rho_g \ln g_{t-1} + \varepsilon_{g,t} \qquad 0 < \rho_g < 1$$

with the shock's variance σ_g^2 .

3.2.4 Equilibrium

In symmetric equilibrium, $\kappa_t(i) = u_t K_t$, $N_t(i) = N_t$, $P_t(i) = P_t = 1$, $Y_t(i) = Y_t$ and $\Pi_t(i) = \Pi_t = Y_t - w_t N_t - r_t u_t K_t$, hold and (3.1) becomes

$$\frac{\lambda - 1}{\lambda} - \phi_t + \mu_t \left[\theta_t \xi_t \frac{\lambda - 1}{\lambda} - \phi_t \right] = 0.$$
(3.2)

From (3.2), and if $\theta_t \xi_t \frac{\lambda-1}{\lambda} < \phi_t < \frac{\lambda-1}{\lambda}$, the financial constraint binds, thus, marginal costs equal

$$\phi_t = \theta_t \xi_t = \tau \theta_t \left(\frac{Y_t}{\bar{Y}_t}\right)^{\gamma}.$$

In the steady state, τ equals marginal costs ϕ , i.e. the inverse of the markup, thus it is not a free parameter.

3.2.5 Self-fulfilling dynamics

The detrended and linearized economy is solved numerically (using standard parameters as listed in Table 3.1). We assume a certain degree of market power such that the credit constraint is always binding, i.e. $\phi_t^{-1} > \frac{\lambda}{\lambda-1}$. Figure 3.2 maps the local dynamics' zones in the $\gamma - \phi^{-1}$ -space. If the credit limit is close to constant, i.e. the parameter γ is small, the economy's dynamics are unique. However, combinations of market power and a procyclical credit limit delivers indeterminacy. The indeterminacy mechanism operates via an upwardly sloping wage-hours locus similar to many animal spirits models.⁵ Then, how can, say, pessimistic expectations about the future create problems? The storyline would go as follows: if people believe that the future is worse, they will attempt to work more hours. In terms of the labor market equilibrium, this change in

 $^{{}^{5}}$ See for example, Farmer and Guo (1994) or Wen (1998).

expectations will shift the labor supply curve outwards. But their pessimistic expectations will also lead households to decrease the lending to firms. This contraction of credit will tighten the firms' borrowing constraints; given the cost structure, the markup will rise and the individual labor demand schedules move leftwards. As a consequence, the economy's wage-hours-locus is upwardly sloping. In equilibrium, the outward shift of labor supply will result in lower employment and in a drop in aggregate production. In sum, the low animal spirits will be self-fulfilling.



Figure 3.2: Parameter spaces of dynamics.

3.3 Estimation

Our next step is to discuss how animal spirits are introduced into the model, to present the data that is employed in the analysis, as well as to outline the full information Bayesian estimation of the artificial economy. We quantify the contribution of animal spirits shocks to business cycle fluctuations. Finally, we compare the estimated shocks to corresponding empirical measures.

3.3.1 Animal spirits in the rational expectations model

If there are many rational expectations equilibria in the model economy, this continuum is a device to introduce animal spirits. In fact, we treat them as quasi-fundamentals as they select from the many possible outcomes. Concretely, we break down the forecast error of output in the linearized model

$$\eta_t^y \equiv \hat{y}_t - E_{t-1}\hat{y}_t$$

(hats denote percentage deviations from steady states) into five fundamental and one non-fundamental components, as suggested by Lubik and Schorfheide (2003):

$$\eta_t^y = \Omega_x \varepsilon_t^x + \Omega_a \varepsilon_t^a + \Omega_\Delta \varepsilon_t^\Delta + \Omega_g \varepsilon_t^g + \Omega_\theta \varepsilon_t^\theta + \varepsilon_t^b.$$

The parameters Ω_x , Ω_a , Ω_{Δ} , Ω_g and Ω_{θ} determine the effect of technological progress, investment-specific technology, preferences, government spending and collateral shocks on the expectations error. This break-down leaves the belief shock ε_t^b as a residual. The last equation then promulgates a strict definition of animal spirits: they are orthogonal to the other disturbances, thus independent of economic fundamentals.

3.3.2 Data and measurement equation

The estimation uses quarterly U.S. data running from 1955: I to 2014: IV and includes seven observable time series: (i) the log difference of real per capita GDP, (ii) real per capita consumption, (iii) real per capita investment, (iv) real per capita government spending, (v) the relative price of investment, (vi) the log difference of per capita hours worked from its sample mean, as well as (vii) the credit spread from its sample mean. We instrument financial market conditions by a credit spread similar to Christiano et al. (2014). In particular, Christiano et al. make use of the difference between the interest rate on Baa corporate bonds and the ten-year US government bond rate. The Appendix provides the full description of the data used and its construction. The corresponding measurement equation is

$$\begin{bmatrix} \ln Y_t - \ln Y_{t-1} \\ \ln C_t - \ln C_{t-1} \\ \ln A_t I_t - \ln A_{t-1} I_{t-1} \\ \ln G_t - \ln G_{t-1} \\ \ln A_t - \ln A_{t-1} \\ \ln N_t - \ln \bar{N} \\ \text{credit spread} \end{bmatrix} = \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + \hat{\mu}_t^y \\ \hat{c}_t - \hat{c}_{t-1} + \hat{\mu}_t^y \\ \hat{i}_t - \hat{i}_{t-1} + \hat{\mu}_t^y \\ \hat{g}_t - \hat{g}_{t-1} + \hat{a}_t^g - \hat{a}_{t-1}^g + \hat{\mu}_t^y \\ \hat{M}_t \\ -x * \phi * \hat{\phi}_t \end{bmatrix} + \begin{bmatrix} \ln \mu^y \\ \ln \mu^y \\ \ln \mu^y \\ \ln \mu^y \\ \ln \mu^a \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} \varepsilon_{y,t}^{me} \\ 0 \\ \eta_t \\ 0 \\ 0 \\ \varepsilon_{s,t}^{me} \end{bmatrix}$$

where $a_t^g \equiv X_t^G/X_t^Y = (a_{t-1}^g)^{\psi_{yg}}(\mu_t^y)^{-1}$. In the last measurement equation, x is the scale parameter only appearing in the measurement equation to adjust the difference of the volatilities (that is, units) between the model frictions and the observable variable. Both output growth and credit spread are measured with errors $\varepsilon_{y,t}^{me}$ and $\varepsilon_{s,t}^{me}$ which are i.i.d. innovations with mean zero and standard deviation σ_y^{me} and σ_s^{me} , respectively. Allowing for a measurement error to output is a way to circumvent stochastic singularity (e.g. Schmitt-Grohé and Uribe, 2012). The measurement error to the spread can reconcile any mis-measurement in the data, especially since only a proxy is observed (e.g. Justiniano et al., 2011). Both measurement errors are restricted to absorb no more than ten percent of the variance of the corresponding observables. We estimate the model by allowing all fundamental and the animal spirits shocks to matter.

3.3.3 Calibrations and priors

We group the model parameters into two categories: calibrated and estimated. The first set of parameters is calibrated following the literature and is based on national accounts data averages. We only address some of these calibrations (all are listed in completion in Table 3.1). The elasticity of substitution parameter λ is set at ten, as in Dotsey and King (2005) and Cogley and Sbordone (2008). The average government spending share in GDP, G/Y, is calibrated at 21 percent, a number which matches national accounts average. The quarterly growth rates of per capita output μ^y and the relative price of investment μ^a are set equal to their sample averages of 1.0041 and 0.9949. Finally, the household's first-order conditions determine the elasticity of the depreciation rate from $\nu = (\mu^k/\beta - 1)/\delta$.

Parameters	Values	Description
β	0.99	Subjective discount factor
α	1/3	Capital share
η	0	Labor supply elasticity parameter
λ	10	Elasticity of substitution between goods
δ	0.0333	Steady-state depreciation rate
u	1	Steady-state capacity utilization rate
G/Y	0.21	Steady-state government expenditure share of GDP
μ^y	1.0041	Steady-state gross per capita GDP growth rate
μ^a	0.9949	Steady-state gross growth rate of price of investment

Table 3.1: Calibration

All other model parameters are estimated. Our prior assumptions are sum-

	Prior	distribution	Posterior distribution		
Estimated parameters	Range	Density[mean,std]	Mean	90% Interval	
Steady-state marginal cost, ϕ	[0.83, 0.90]	Beta[0.88,0.01]	0.833	[0.831, 0.834]	
Elasticity of collateral, γ	[0.160, 0.607]	Uniform	0.322	[0.315, 0.329]	
Gov. trend smoothness, ψ_{yq}	[0,1)	Beta[0.5, 0.2]	0.965	[0.953, 0.977]	
Scale parameter, x	R^+	IGam[44, Inf]	47.33	[44.28, 50.46]	
AR technology shock, ρ_x	[0,1)	Beta[0.5, 0.2]	0.025	[0.008, 0.041]	
AR investment shock, ρ_a	[0,1)	Beta[0.5, 0.2]	0.029	[0.013, 0.045]	
AR preference shock, ρ_{Δ}	[0,1)	Beta[0.5, 0.2]	0.984	[0.981, 0.988]	
AR government shock, ρ_g	[0,1)	Beta[0.5, 0.2]	0.986	[0.982, 0.989]	
AR collateral shock, ρ_{θ}	[0,1)	Beta[0.5, 0.2]	0.992	[0.990, 0.994]	
Belief shock volatility, σ_b	R^+	IGam[0.1,Inf]	0.640	[0.615, 0.665]	
SE technology shock, σ_x	R^+	IGam[0.1,Inf]	0.690	[0.646, 0.733]	
SE investment shock, σ_a	R^+	IGam[0.1,Inf]	0.562	[0.525, 0.598]	
SE preference shock, σ_{Δ}	R^+	IGam[0.1,Inf]	0.386	[0.364, 0.407]	
SE government shock, σ_g	R^+	IGam[0.1,Inf]	0.944	[0.896, 0.992]	
SE collateral shocks, σ_{θ}	R^+	IGam[0.1,Inf]	0.132	[0.121, 0.143]	
SE measurement error, σ_y^{me}	[0, 0.29]	Uniform	0.290	[0.289, 0.290]	
SE measurement error, σ_s^{me}	[0,27.42]	Uniform	27.28	[27.11, 27.42]	
Technology shock effect, Ω_x	[-3,3]	Uniform	-0.514	[-0.590, -0.438]	
Investment shock effect, Ω_a	[-3,3]	Uniform	0.271	[0.176, 0.367]	
Preference shock effect, Ω_{Δ}	[-3,3]	Uniform	0.872	[0.756, 0.994]	
Government shock effect, Ω_g	[-3,3]	Uniform	0.256	[0.205, 0.305]	
Collateral shock effect, Ω_{θ}	[-3,3]	Uniform	0.999	[0.610, 1.393]	
Log-data density		4064.98			

Table 3.2: Estimation

marized in Table 3.2. The parameters estimated here include the steady state marginal cost ϕ (or equivalently the inverse of the mark-up), the elasticity of collateral γ , the scale parameter x, the parameters that describe the stochastic processes and the standard deviation of the measurement error. A beta distribution is adopted for the steady-state marginal cost ϕ and its value falls between 0.83 and 0.9, so that the steady-state markup varies from around eleven to twenty percent. The range of marginal costs is chosen for two reasons. First, the empirically estimated markup falls in this range (see for example Cogley and Sbordone, 2008, and De Loecker and Eeckhout, 2017). Second, the upper value of ϕ is further restricted by the inequality constraints $\xi \frac{\lambda-1}{\lambda} < \phi < \frac{\lambda-1}{\lambda}$ for the financial constraint to bind.⁶ We set the prior mean for x to match the standard

⁶The prior distribution of γ guarantees that the complete indeterminacy region is covered. Since we concentrate on this region, during the MCMC, all proposed draws from the

deviation of the smoothed endogenous financial frictions in the model without any financial information (data and shock) and the standard deviation of the demeaned spread data. We adopt an inverse gamma distribution for the prior. For the persistence parameters we use a beta distribution and the standard deviations of the shocks follow an inverse gamma distribution. The prior distributions for the expectational parameters Ω_x , Ω_a , Ω_Δ , Ω_g and Ω_θ are uniform, thus agnostic about their values. Endogenous priors prevent overpredicting the model variances as in Christiano et al. (2011). We use the Metropolis-Hastings algorithm to generate one million draws from the posterior for each of the two chains, discard the initial half of the draws as burn-in, and adjust the scale in the jumping distribution to achieve a 25 to 30 percent acceptance rate for each chain.

3.3.4 Estimation results

The last two columns of Table 3.2 present the posterior means of the estimated parameters, along with their 90 percent posterior probability intervals. The parameters are precisely estimated as is evidenced by the percentiles. The estimated steady state of marginal cost implies a steady state markup of twenty percent. The table also reveals a significantly time-varying character of financial frictions. Disturbances to preference, government spending and collateral exhibit a high degree of persistence. The autocorrelation of the non-stationary technology shock is low, but it is not inconsistent with the moderate values commonly found in the literature.

determinacy and source regions were discarded.

		Data			Model	
x	σ_x/σ_Y	$\rho(x,Y)$	ACF	σ_x/σ_Y	$\rho(x,Y)$	ACF
Y_t	1.00	1.00	0.93	1.00	1.00	0.91
C_t	0.58	0.85	0.92	0.63	0.75	0.90
I_t	3.25	0.89	0.94	3.09	0.88	0.92
G_t	0.99	0.01	0.94	0.96	0.21	0.90
N_t	1.24	0.87	0.94	1.01	0.98	0.92

Table 3.3: Business cycle dynamics (band-pass filtered)

Table 3.3 reports second moments of the main macroeconomic variables calculated using U.S. data and compares these moments to those obtained from model simulations at the posterior mean, both at business cycle frequencies. The model matches fairly well the relative standard deviations, autocorrelations and the variables' cross-correlations with output. Table 3.4 displays the contribution of each structural shock, which we list in the top row, to the variances of key macroeconomic variables. Through the lens of our theory, the decomposition suggests that animal spirits shocks ε_t^b are a major source of U.S. aggregate fluctuations. These shocks account for over 40 percent of output growth fluctuations. The ensemble of other aggregate demand shocks plays a lesser role and the contribution of the two technology shocks is small at no more than twenty percent. For investment, the vast majority of its variations comes from animal spirits suggesting that much of the spending is driven by entrepreneurial sentiments. The credit spread is mainly driven by stochastic financial factors as well as by the three demand side disturbances (i.e. animal spirits, preferences and government spending).⁷ We re-ran the estimation, but halted in 2007:III,

⁷We estimate the model using loan data and animal spirits remain significant. Furthermore,

i.e. just before the onset of the Great Recession. This alteration does not affect the results as the parameter estimates as well as the variance decompositions remain virtually unchanged.

Series/shocks	ε^b_t	ε_t^x	ε^a_t	ε^{Δ}_t	ε^g_t	ε^{θ}_t	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln{(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1})}$	43.43	11.17	5.72	15.70	9.93	6.71	6.80	0.00
$\ln{(\boldsymbol{C}_t/\boldsymbol{C}_{t-1})}$	6.18	40.42	2.76	39.84	1.96	8.82	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	66.53	2.41	7.06	9.34	7.09	7.57	0.00	0.00
$\ln{(N_t/\bar{N})}$	21.24	2.54	9.37	26.50	22.06	18.30	0.00	0.00
$\ln{(G_t/G_{t-1})}$	0.00	0.98	0.16	0.00	98.85	0.00	0.00	0.00
$\ln{(A_t/A_{t-1})}$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	12.26	2.06	4.85	17.99	15.06	43.49	0.00	3.30

Table 3.4: Unconditional variance decomposition

In sum, the estimation suggests that psychological motivations are behind a significant portion of the fluctuations in U.S. aggregate real economic activity. While the definitions of confidence shocks do not exactly overlap, this result parallels recent findings by Angeletos et al. (2016), Milani (2017) and Nam and Wang (2016) who, while arguing within theoretical frameworks that involve uniqueness, also find that bouts of optimism and pessimism are driving a large fraction of U.S. aggregate fluctuations.

3.3.5 Are shocks meaningfully labeled?

We identify the shocks by estimating in a system and it is thus fair to ask if the estimated shocks are meaningfully labelled. Specifically, do the shocks share resemblance with empirical series that are computed with orthogonal information variance decompositions at business cycle frequencies deliver almost identical results.



Figure 3.3: Fernald's vs model's total factor productivity (annual data).

sets? To begin with, the estimated model's total factor productivity (TFP) series is compared with Fernald's (2014) TFP series for the United States.⁸ Fernald's TFP series are widely considered as the gold standard for this variable for which he adjusts for variations in factor utilization (labor effort and the workweek of capital) as well as labor skills. The results of this exogenous validation are reassuring as shown in Figure 3.3. Both productivity series not only have similar amplitudes, but their contemporaneous correlation comes in at 0.68. Hence, the model is successful in extracting productivity shocks. Next, Figure 3.4 compares the index of estimated confidence and the U.S. Business Confidence index (bandpass filtered to concentrate on the relevant frequencies). Clearly, the empirical confidence index is influenced by a raft of fundamentals and non-fundamentals, thus, it is not exactly clear how the empirical data would map our theoretical notion of animal spirits. Yet, one would expect that the animal spirits and confidence data display a certain similarity. In fact, the two sentiment series are strongly correlated and we interpret the relationship in Figure 3.4 as endorsing

⁸Growth of total factor productivity in our model is given by $(1 - \alpha)(\hat{\mu}_t^x + \ln \mu^x)$.

our estimation and as supporting the case that estimated belief shocks reflect variations in people's expectations about the future path of the economy.⁹



Figure 3.4: Business confidence index vs animal spirits shocks (normalized data).

3.4 Robustness checks

In this Section, we report several robustness checks. First, we leave Lubik and Schorfheide's (2003) representation of a belief shock and follow Farmer et al.'s (2015) formulation. Next, we go through alternative observables to measure financial markets' health. This is followed by adding Fernald's (2014) TFP data to the observables. We also replace permanent technology shocks by transitory shocks and consider the presence of shocks to the marginal efficiency of investment as in Justiniano et al. (2011).

We begin the chain of robustness checks by following the approach of Farmer et al. (2015) in which the animals spirits shock is simply the forecast error, i.e. $\eta_t^y = \varepsilon_t^b$, with a variance σ_{η}^2 . Intuitively, since output is forward looking,

 $^{^9\}mathrm{The}$ correlation of the estimated sunspot shocks and Fernald's TFP series is insignificant at 0.2.

	Model with $\eta_t^y = \varepsilon_t^b$							
	Prior	distribution	Posteri	or distribution				
Parameters	Range	Density[mean,std]	Mean	90% Interval				
ϕ	[0.83, 0.90]	Beta[0.88,0.01]	0.833	[0.831, 0.834]				
γ	[0.160, 0.607]	Uniform	0.322	[0.315, 0.329]				
$\psi_{oldsymbol{uq}}$	[0,1)	Beta[0.5, 0.2]	0.965	[0.954, 0.977]				
x	R^+	IGam[44,Inf]	47.30	$[44.18,\!50.35]$				
$ ho_x$	[0,1)	Beta[0.5, 0.2]	0.025	[0.008, 0.042]				
$ ho_a$	[0,1)	Beta[0.5, 0.2]	0.029	[0.014, 0.045]				
$ ho_\Delta$	[0,1)	Beta[0.5, 0.2]	0.984	$[0.981, \! 0.988]$				
$ ho_{a}$	[0,1)	Beta[0.5, 0.2]	0.986	[0.982, 0.989]				
$\rho_{ heta}$	[0,1)	Beta[0.5, 0.2]	0.992	[0.990, 0.994]				
σ_η	R^+	IGam[0.1,Inf]	0.862	[0.821, 0.902]				
σ_x	R^+	IGam[0.1,Inf]	0.690	[0.647, 0.733]				
σ_a	R^+	IGam[0.1,Inf]	0.562	[0.525, 0.598]				
σ_{Δ}	R^+	IGam[0.1,Inf]	0.385	[0.364, 0.407]				
σ_{g}	R^+	IGam[0.1,Inf]	0.945	[0.897, 0.993]				
$\sigma_{ heta}$	R^+	IGam[0.1,Inf]	0.132	[0.121, 0.143]				
σ_u^{me}	[0, 0.29]	Uniform	0.290	[0.289, 0.290]				
σ_s^{me}	[0,27.42]	Uniform	27.28	[27.11, 27.42]				
$\rho(x, \eta^y)$	[-1,1]	Uniform	-0.406	[-0.465, -0.349]				
$\rho(a,\eta^y)$	[-1,1]	Uniform	0.172	[0.110, 0.233]				
$\rho(\Delta, \eta^y)$	[-1,1]	Uniform	0.388	[0.338, 0.438]				
$\rho(g,\eta^y)$	[-1,1]	Uniform	0.275	[0.226, 0.327]				
$ ho(heta,\eta^y)$	[-1,1]	Uniform	0.151	[0.091, 0.213]				
Log-data density	-	4066.02						

Table 3.5: Posterior distribution comparison

this expectation error should be correlated with fundamental shocks. Yet, it is also a sunspot shock, as it can cause movements in economic activity without any shifts to fundamentals. Assuming a uniform distribution, we thus estimate the correlations between η_t^y and the fundamental shocks. The priors for the other parameters are kept the same as in the baseline model. As can be seen by comparing Tables 3.2 and 3.5, our estimation results are robust to the formation of the expectation error. The posterior distributions are almost identical and the closeness of the log-data densities confirms that the goodness of fit between

Series/shocks	ε^b_t	ε_t^x	ε^a_t	ε_t^{Δ}	ε_t^g	ε^{θ}_t	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln\left(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1}\right)$	45.46	11.34	5.34	15.63	9.12	6.31	6.80	0.00
$\ln\left(C_t/C_{t-1}\right)$	6.67	41.08	2.65	38.98	1.84	8.78	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	68.22	2.32	6.45	9.04	6.24	7.73	0.00	0.00
$\ln\left(N_t/\bar{N}\right)$	23.25	2.31	9.08	25.25	20.31	19.79	0.00	0.00
$\ln\left(G_t/G_{t-1}\right)$	0.00	1.07	0.17	0.00	98.76	0.00	0.00	0.00
$\ln\left(A_t/A_{t-1}\right)$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	13.12	1.87	4.59	16.51	13.48	47.13	0.00	3.30

Table 3.6: Unconditional variance decomposition (Baa-Aaa spread)

the models is equivalent.¹⁰

The next robustness check concerns the choice of the observed spread when instrumenting financial markets' conditions as we consider the sensitivity to using various alternative spreads. In particular, we ask if using the Baa-Aaa spread or the Baa-Federal funds rate spread leads to significantly different results in the estimation. We report the variance decompositions only. The results for the alternative spreads are documented in Tables 3.6 and 3.7. Animal spirits continue to stand out as the main driver of the business cycle.¹¹

Series/shocks	ε^b_t	ε_t^x	ε^a_t	ε^{Δ}_t	ε^g_t	ε^{θ}_t	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln{(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1})}$	42.35	12.38	6.10	17.45	9.40	4.97	7.34	0.00
$\ln{(C_t/C_{t-1})}$	5.93	43.61	3.01	39.50	1.86	6.09	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	65.43	2.62	7.51	10.04	7.00	7.40	0.00	0.00
$\ln{(N_t/\bar{N})}$	22.11	2.33	10.53	26.72	22.55	15.76	0.00	0.00
$\ln{(G_t/G_{t-1})}$	0.00	1.02	0.17	0.00	98.81	0.00	0.00	0.00
$\ln{(A_t/A_{t-1})}$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	14.32	2.19	6.08	20.38	17.16	34.61	0.00	5.26

Table 3.7: Unconditional variance decomposition (Baa-FF spread)

 $^{10}{\rm Second}$ moments and variance decompositions are virtually identical and are not presented to conserve space.

¹¹We considered other interest spreads and the results repeat.

Next, we add total factor productivity to the catalog of observables. Fernald's (2014) data is the natural series to choose from. Fernald adjusts for variations in factor utilization (labor and capital) and includes adjustment for quality or composition of inputs. Most of these influences are not part of the present artificial economy and we thus add one more measurement error on total factor productivity (at not more than ten percent). Table 3.8 shows that the previous results remain robust. Animal spirits continue to cause the bulk of U.S. output fluctuations. The technology shocks' contributions are lower, with a best point estimate near ten percent.

Table 3.8: Unconditional variance decomposition (Fernald TFP)

Series/shocks	ε^b_t	ε_t^x	ε^a_t	ε^{Δ}_t	ε_t^g	ε^{θ}_t	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$	$\varepsilon_{tfp,t}^{me}$
$\ln{(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1})}$	39.02	10.35	5.10	12.63	9.13	17.01	6.77	0.00	0.00
$\ln{(\boldsymbol{C}_t/\boldsymbol{C}_{t-1})}$	4.63	38.01	2.18	34.21	1.49	19.48	0.00	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	59.56	2.09	6.31	8.56	6.12	17.36	0.00	0.00	0.00
$\ln{(\boldsymbol{N}_t/\bar{N})}$	16.00	2.35	7.14	21.74	16.70	36.07	0.00	0.00	0.00
$\ln{(G_t/G_{t-1})}$	0.00	1.08	0.15	0.00	98.76	0.00	0.00	0.00	0.00
$\ln{(A_t/A_{t-1})}$	0.00	0.00	100.00	0.00	0.00	0.00	0.00	0.00	0.00
Credit spread	6.54	1.34	2.61	10.38	8.13	67.28	0.00	3.71	0.00
$\ln{(TFP_t/TFP_{t-1})}$	0.00	92.29	0.00	0.00	0.00	0.00	0.00	0.00	7.71

So far, we have assumed that technology follows a stochastic trend. We now replace permanent technology shocks by transitory shocks. Hence, the production technology is given by

$$Y_t = Z_t K_t^{\alpha} (\mu^t N_t)^{1-\alpha}$$

and the growth rate of labor augmenting technological progress is deterministic at the constant rate μ , as in King et al. (1988). We permit temporary changes in total factor productivity through Z_t , which follows a first-order autoregressive process

$$\ln Z_t = (1 - \rho_z) \ln Z + \rho_z \ln Z_{t-1} + \varepsilon_{z,t} \qquad 0 < \rho_z < 1.$$

The presence of (one more) transitory shock will also make it (even) harder for animal spirits shocks to explain data's transitory fluctuations. Nevertheless, the model estimation delivers similar posterior means of the parameters as the baseline estimation and they are reported in the Appendix. Noteworthy is the estimate for ρ_z at 0.997 which is arguably very close to a unit root. While high, this number is consistent with Ireland (2001), for example. The variance decompositions of the stationary technology shocks model are reported in Table 3.9. Technology shocks account for about 17 percent of GDP volatility. Animal spirits remain the most critical driver of aggregate fluctuations and they continue to explain roughly 40 percent of output growth variations.¹²

Series/shocks	ε^b_t	ε_t^z	ε^a_t	ε^{Δ}_t	ε^g_t	ε^{θ}_t	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln{(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1})}$	39.18	16.79	5.28	15.64	8.21	8.69	6.22	0.00
$\ln{(\boldsymbol{C}_t/\boldsymbol{C}_{t-1})}$	3.78	43.19	2.19	40.98	1.13	8.73	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	57.92	11.81	6.28	10.25	5.64	8.10	0.00	0.00
$\ln{(N_t/\bar{N})}$	16.08	17.47	8.35	26.25	15.10	16.75	0.00	0.00
$\ln{(G_t/G_{t-1})}$	0.00	0.00	0.22	0.00	99.78	0.00	0.00	0.00
$\ln{(A_t/A_{t-1})}$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	5.63	41.34	2.59	10.61	6.17	30.37	0.00	3.30

Table 3.9: Unconditional variance decomposition (transitory TFP)

¹²The posterior means of the parameters in the model with transitory technology productivity are shown in the Appendix as Table 3.13. There, we also report an external validation as in Figures 3.3 and 3.4 and, again, estimated shocks are very similar to Fernald's series as well as U.S. confidence data.

The natural question arises which specification of technology is favored by data? This question is answered in Table 3.10 which compares the model fits of the two alternatively specified models. Data strongly prefers a version of the model in which total factor productivity has a stochastic trend.¹³

Table 3.10: Model comparison

	Baseline: permanent TFP	Alternative: transitory TFP
Log-data density	4064.98	3811.89

Justiniano et al. (2011) push for shocks that affect the production of installed capital from investment goods or the transformation of savings into the future capital input. This is an alternative way to model exogenous financial frictions. The concept of shocks to the marginal efficiency to investment (MEI) goes back to Greenwood et al. (1988) who formulate the ideas as

$$K_{t+1} = (1 - \delta_t)K_t + \nu_t I_t$$

where we abstract from adjustment costs to not mess with the indeterminacy properties of the artificial economy. The shock ν_t affects the marginal efficiency of capital and it follows an autoregressive process with persistence parameter ρ_{ν} . The MEI shocks are likely a

"might proxy for more fundamental disturbances to the intermediation ability of the financial system." [Justiniano et al., 2011, 103]

We estimate the amended model and associate the observed spread with the value of the MEI to impose discipline on the inference of the shock as in

¹³We conduct a similar exercise with respect to the form of the preference shock. Data does strongly prefer the current setup over a version with a stochastic discount factor.

Justiniano et al. (2011).¹⁴ Again, we add a measurement error to the spread equation. Table 3.11 shows, in line with our previous findings, that the animal spirits shocks remain a most prominent driver of U.S. output fluctuations.¹⁵ An external validation exercise akin to Figures 3.3 and 3.4 finds that estimated shocks are again very similar to their empirical counterparts (see Appendix).

Series/shocks	ε^b_t	ε_t^x	ε^a_t	ε^{Δ}_t	ε_t^g	ε_t^{MEI}	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln{(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1})}$	46.82	10.15	5.51	15.76	11.18	2.08	8.49	0.00
$\ln{(C_t/C_{t-1})}$	8.77	40.93	2.92	43.77	2.96	0.66	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	69.61	2.35	6.77	9.82	8.68	2.77	0.00	0.00
$\ln{(N_t/\bar{N})}$	25.57	3.62	10.02	31.30	27.17	2.31	0.00	0.00
$\ln{(G_t/G_{t-1})}$	0.00	0.75	0.13	0.00	99.12	0.00	0.00	0.00
$\ln\left(A_t/A_{t-1}\right)$	0.00	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	0.00	0.00	0.00	0.00	0.00	99.95	0.00	0.05

Table 3.11: Unconditional variance decomposition (MEI shock)

3.5 A closer look at the Great Recession

From 2007 to 2009, the U.S. economy was in a severe slump. The Great Recession was the single-worst economic contraction since the 1930s, with economic activity diving after various financial institutions collapsed. One of the aims of the recent financial friction models is to identify the sources of the crisis.

¹⁴Given the occurance of financial frictions in two places, we are only able to connect one model friction to the spread's measurement equation. The series of animal spirits remains highly correlation to earlier estimations, thus, our result is not the consequence of putting less restictions on the psychological shocks.

¹⁵We considered the hypothesis that sunspot shocks are in fact news shocks. In the spirit of Beaudry and Portier (2006), we looked into finding a relation of the belief shocks with future movements of technology. In particular, we compute the correlations of the estimated animal spirits with Fernald's TFP data at four to sixteen quarters out. The correlations are negligible at never more than 0.04.

To what extent can animal spirits explain the downturn in GDP observed in this recession?



Figure 3.5: Counterfactual path of output, conditional on estimated belief shocks. Parameters are set at the posterior mean.

We begin with a counterfactual exercise in which we shut down all but the animal spirits shocks (using Section 3.3's model). Figure 3.5 plots the counterfactual path of output driven solely by these belief shocks along with the actual series over the Great Recession period. The U.S. data has been detrended by removing long-run productivity trend and also population growth, as we abstract from it in the model. We re-scale both model and U.S. data so that outputs are equal to 100 in 2008:I. The model economy virtually coincides in both timing and depth with the actual economy during the crisis period and the measured drop in confidence can account for most of the decline in output. The counterfactual exercise favors the interpretation that the fall of aggregate output during the Great Recession was closely associated with self-fulfilling beliefs. Our reading of events goes like this: adverse expectations led to a drop in aggregate demand which curbed lending and tightened credit (similar to Kahle and Stulz, 2013). This tightening occurred because people were expecting worsening business conditions and higher defaults. In other words, people became pessimistic and, as a consequence of the effect on financial markets, the reduced investment spending lowered productivity which then made pessimistic expectations self-fulfilled. Our results do not necessarily contradict Christiano et al.'s (2015) account of the Great Recession. Their study finds that the steep decline of aggregate economic activity was overwhelmingly caused by exogenous financial frictions. What our analysis suggests is, however, that it was a drop in people's animal spirits affected aggregate demand and then found its catalyst in financial markets. The endogenous reaction of the financial sector helped in propagating gloomy animal spirits into the full-blown crisis and macroeconomic collapse.



Figure 3.6: The artificial labor wedge during the Great Recession.

A useful way of thinking about the Great Recession is in terms of Chari et al.'s (2007) business cycle accounting framework which decomposes distortions in the economy into sets of residuals or wedges. When applying this framework, Brinca et al. (2016) assert that

"[...] considering the period from 2008 until the end of 2011, [our] results imply that the Great Recession in the United States should be thought of as primarily a labor wedge recession, with an important secondary role for the investment wedge." [Brinca et al., 2016, 1042]

This diagnostic finding leads to the question of what would the these wedges look like in the artificial economy? In a benchmark prototype economy, the labor wedge $1 - \tau_t^n$ shows up in the budget constraint as

$$\dots = (1 - \tau_t^n) w_t N_t + r_t u_t K_t$$

thus it is like a tax on labor services.¹⁶ The labor wedge is plotted along with its data equivalent in Figure 3.6. Clearly, the two series show high conformity. The artificial wedge explains about three-fourths of the data wedge's plunge during 2008 and 2009 and it charts a tepid recovery over the 2010 to 2014 period. Our model estimation also suggests an important role for financial market imperfections. Thus, given Brinca et al.'s (2016) assertion, we report a wedge that measures these sort of distortions: it is like a tax on capital income as in Kobayashi and Inaba (2006) or Cavalcanti et al. (2008) and in a benchmark prototype economy it would show up on the right hand side of the budget constraint as $1 - \tau_t^k$:

$$\dots = w_t N_t + (1 - \tau_t^k) r_t u_t K_t$$

Figure 3.7 maps out both the empirical and the model implied capital wedges next to the investment wedge as in Brinca et al. (2016). Note that we report the " τ_t s" rather than the full wedges. These distortions are shown alongside Romer and Romer's (2017) semi-annual index of financial stress which focusses

"on disruptions to credit supply, rather than on broader concep-

¹⁶In the Appendix, we describe the construction of wedges in terms of the artificial economy. Kobayashi and Inaba (2006) prove an equivalence of the capital wedge as well as the investment wedge.

tions of financial problems" [Romer and Romer, 2017, 3073].

We take three insights from this accounting. Firstly, capital and investment wedges display very similar patterns and they indeed point to a worsening of financial market health after 2007. This mirrors Romer and Romer's (2017) findings. Second, our model lines up well with Brinca et al.'s (2016) interpretation of the Great Recession in terms of both the labor as well as financial wedges. Thirdly, Romer and Romer's (2017) index suggests that financial distress in the U.S. ended by 2011 and this is at some odds with the pattern of both financial wedges which are significantly more persistent. Our take on this picture is that investment spending remained subdued for factors other than financial ones. From our analysis, it appears that the tepid spending reflects a lack of animal spirits, i.e. businesses were not confident about future demand to justify more investment.



Figure 3.7: Financial wedges during the Great Recession: the initial observations have been normalized to 100 (capital wedges measured on left-hand axis). Right-hand panel shows Romer and Romer (2017) index.

3.6 Does data prefer indeterminacy?

So far we have restricted the estimation to the parameter space with multiple equilibria, yet a natural question arises: does data in fact favor a model with indeterminacy? To answer this question, we now estimate the economy over the entire parameter space using the methodology proposed in Bianchi and Nicolò (2017).¹⁷ Their procedure can be implemented without knowing the analytical expressions for the boundaries between the three dynamic regions (recall Figure 3.2).

	Determinacy	Indeterminacy
Model prior probabilities	0.52	0.47
Permanent TFP		
Log-data density	3470.07	4065.42
Model posterior probability	0.00	1.00
Transitory TFP		
Log-data density	3441.67	3812.86
Model posterior probability	0.00	1.00
MEI		
Log-data density	3601.05	4305.71
Model posterior probability	0.00	1.00

 Table 3.12: Determinacy versus Indeterminacy

The estimation process begins by setting the priors so that determinacy, indeterminacy and source probabilities are at 52:47:1 (in percent). To do this, we adjust the prior of the elasticity of the collateral γ , which is now beta-distributed, to being centered at 0.17 with a standard deviation of 0.1 and truncated to be no more than 0.61.¹⁸ All parameters that pertain to the solution under indeterminacy are restricted to be zero when the estimation for draws is taking

¹⁷The Appendix explains their methodology in more detail.

¹⁸All other priors are as above. Details of the estimation procedure are delegated to the Appendix 3.A.3.

place in the determinacy region of the model. Draws from the source region were discarded. In line with Bianchi and Nicolò (2017), we follow the approach proposed in Farmer et al. (2015) and construct the forecast errors of output η_t^y as a belief shock with variance σ_{η}^2 and allow the expectation errors to be correlated with the fundamental shocks. As would be reasonable, for these correlations we assume flat priors that are uniform between -1 and 1. Table 3.12 presents the results for model versions discussed earlier involving i) permanent technology shocks, ii) transitory technology shocks and iii) shocks to the marginal efficiency to investment. The observable variables are the same as in Sections 3.3 and 3.4. The log data densities in Table 3.12 suggest that U.S. data strongly favours the indeterminacy model over all three versions of the economy in which animal spirits cannot play a role.

Three further observations are worthwhile mentioning. First, the estimated parameters under indeterminacy that arise when we implement the methodology developed in Bianchi and Nicolò (2017) are essentially equivalent to our previous results. Thus, estimating via their procedure leaves results unaffected and the implications regarding the important role of animal spirits carry over (see for example Table 3.14 in the Appendix). Second, in addition to being favored by data, the indeterminacy model is superior in identifying shocks for which empirical counterparts exist. For example, the model-based technology shocks track the empirical TFP series better under indeterminacy: when comparing the estimated sequence as done in the external validation of Figure 3.3, then the contemporaneous correlation with Fernald's series drops slightly from 0.68 to 0.65 under determinacy. Third, the key difference in the parameter estimates across the two regions applies to the parameter γ that controls the endogenous component of credit market tightness: γ approaches zero for the determinacy versions

of the model. The endogeneous aspect of the collateral constraint disappears.

How can we make sense of the finding that the indeterminacy model is preferred by U.S. data? The absence of the endogenous feedback of financial market conditions to the state of the economy implies that other fundamental shocks' amplification mechanisms are curtailed and movements of the collateral constraint (and of marginal costs) are determined by the exogenous financial friction shocks. For example, as is shown in the Appendix' Table 3.15, under determinacy the MEI shock explains about thirty percent of output fluctuations and the spread's variations in almost their entirety. These numbers are quite similar to Justiniano et al. (2011, Table 4) while at somewhat different frequencies. However, the rigid collateral constraints imply that the other fundamental shocks are no longer able to contribute towards the procyclical variations of financial health. In other words, the pattern that was reported in Figure 3.1 namely that financial conditions are cyclical and deteriorate during basically all slumps – is more effortlessly accommodated by an artificial economy with an endogenously varying collateral constraint, however, this then implies that the economy becomes indeterminate and, consequently, animal spirits are assigned an important role.

3.7 Concluding remarks

This paper has presented evidence on the sources of U.S. aggregate fluctuations over the period 1955 to 2014. We perform a Bayesian estimation of a financial accelerator model which features an indeterminacy of rational expectations equilibria. Indeterminacy in the model is linked to the empirically observed countercyclical movement of financial market tightness. The artificial economy is driven both by fundamental shocks as well as by animal spirits. U.S. data favours
the indeterminacy model over versions of the economy in which sunspots do not play a role. The estimation supports the view that people's animal spirits play a significant role for the U.S. business cycle. Variance decompositions suggest that animal spirits are behind a substantial fraction of output growth variations and they explain an even larger portion of fluctuations in investment spending. Technology shocks and financial frictions shocks are significantly less important in explaining the oscillations in aggregate real economic activity. The 2007-2009 recession appears to have been chiefly caused by adverse confidence shocks.

Admittedly, we have left out various aspects of the economy that could be considered relevant. For example, the economy is real and nominal variables are absent. Thus, we exclude the potential effects of price stickiness and any influence of a monetary authority. Also, the absence of monetary policy as well as the exogenous character of the fiscal side precludes from addressing how policy could potentially influence the dynamics of this economy. The smallscale character of our model, however, provides the advantage of tractability specifically when conducting the various robustness exercises. This being said, mentioned extensions are beyond the scope and the goals of the current paper, but we plan to work out a medium-scale version of the indeterminacy model in the future.

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3.A Appendix

The Appendix sets out the complete model, a discussion of the wedges, and it lists the data sources and definitions. We begin with collecting the model's equations.

3.A.1 Model equations and equilibrium dynamics

The first-order conditions for the household's optimization problems are

$$\varphi N_t^\eta = \frac{1}{C_t - \Lambda_t} W_t$$

$$r_t = A_t \delta_0 u_t^{\nu}$$

and

$$\frac{A_t}{C_t - \Lambda_t} = \beta E_t \left[\frac{1}{C_{t+1} - \Lambda_{t+1}} (r_{t+1}u_{t+1} + A_{t+1}(1 - \delta_{t+1})) \right].$$

In the model, output, consumption, and real wage fluctuate around the same stochastic growth trend $X_t^Y = X_t A_t^{\alpha/(\alpha-1)}$, the growth rate of which is $\mu_t^y \equiv$

 $X_t^Y/X_{t-1}^Y = \mu_t^x(\mu_t^a)^{\frac{\alpha}{\alpha-1}}$. The trend in capital stock, which is also the trend in investment equals $X_t^K = X_t^Y/A_t$, the growth rate of which is $\mu_t^k \equiv X_t^K/X_{t-1}^K = \mu_t^x(\mu_t^a)^{\frac{1}{\alpha-1}}$. Besides, the government expenditure fluctuates around its own trend X_t^G . There is no growth trend in hours, utilization and marginal cost. We first derive the detrended dynamic equilibrium equations and then log-linearly approximate them around the deterministic steady state. Let $y_t = Y_t/X_t^Y$, $c_t = C_t/X_t^Y$, $w_t = W_t/X_t^Y$, $i_t = I_t/X_t^K$, $k_t = K_t/X_{t-1}^K$, $g_t = G_t/X_t^G$, $\Delta_t = \Lambda_t/X_t^Y$ and y_t/\bar{y} approximately equal to Y_t/\bar{Y}_t , where \bar{y} represents the steady state of detrended output. The log-linearized system is summarized by

$$\hat{y}_t = \alpha \hat{k}_t + \alpha \hat{u}_t - \alpha \hat{\mu}_t^k + (1 - \alpha) \hat{N}_t$$

$$\hat{y}_{t} = \left[1 - \frac{\alpha\phi(\mu^{k} - 1 + \delta)}{\delta(1 + \nu)} - \frac{G}{Y}\right]\hat{c}_{t} + \frac{\alpha\phi(\mu^{k} - 1 + \delta)}{\delta(1 + \nu)}\hat{i}_{t} + \frac{G}{Y}(\hat{a}_{t}^{g} + \hat{g}_{t})$$
$$\hat{y}_{t} = (1 + \eta)\hat{N}_{t} + \hat{c}_{t} - \hat{\Delta}_{t} - \hat{\phi}_{t}$$
$$\hat{y}_{t} = (1 + \nu)\hat{u}_{t} + \hat{k}_{t} - \hat{\phi}_{t} - \hat{\mu}_{t}^{k}$$
$$\hat{k}_{t+1} = \frac{(1 - \delta)}{\mu^{k}}(\hat{k}_{t} - \hat{\mu}_{t}^{k}) + \frac{(\mu^{k} - 1 + \delta)}{\mu^{k}}\hat{i}_{t} - \frac{\delta(1 + \nu)}{\mu^{k}}\hat{u}_{t}$$
$$\hat{c}_{t+1} = \hat{c}_{t} - \hat{\Delta}_{t} - \left[1 - \frac{\beta\delta(1 + \nu)}{\mu^{k}}\right]\hat{\mu}_{t+1}^{k} + \hat{\Delta}_{t+1} + \frac{\beta\delta(1 + \nu)}{\mu^{k}}(\hat{y}_{t+1} - \hat{k}_{t+1} + \hat{\phi}_{t+1} - \hat{u}_{t+1})$$

and

$$\hat{\phi}_t = \gamma \hat{y}_t + \hat{\theta}_t.$$

In these equations, variables without time subscripts refer to steady state values while the hatted variables denote percent deviations from their corresponding steady-state, e.g., $\hat{y}_t \equiv \log(y_t/\bar{y})$. The last equation shows that if $\gamma \to 0$, then marginal cost and the credit constraint are determined by the exogenous financial shocks only. The following table shows the estimation results for transitory technology shocks.

	Prior	distribution	Posterior distribution		
Estimated parameters	Range	Density[mean,std]	Mean	90% Interval	
Steady-state marginal cost, ϕ	[0.83, 0.90]	Beta[0.88, 0.01]	0.832	[0.831, 0.833]	
Elasticity of collateral, γ	[0.160, 0.607]	Uniform	0.296	[0.291, 0.301]	
Gov. trend smoothness, ψ_{yg}	[0,1)	Beta[0.5, 0.2]	0.953	[0.932, 0.975]	
Scale parameter, x	R^+	IGam[44,Inf]	44.38	[42.62, 46.24]	
AR technology shock, ρ_z	[0,1)	Beta[0.5, 0.2]	0.997	[0.996, 0.998]	
AR investment shock, ρ_a	[0,1)	Beta[0.5, 0.2]	0.020	[0.008, 0.032]	
AR preference shock, ρ_Δ	[0,1)	Beta[0.5, 0.2]	0.979	[0.974, 0.983]	
AR government shock, ρ_g	[0,1)	Beta[0.5, 0.2]	0.981	[0.976, 0.987]	
AR collateral shock, ρ_θ	[0,1)	Beta[0.5, 0.2]	0.992	[0.991, 0.994]	
Belief shock volatility, σ_b	R^+	IGam[0.1,Inf]	0.662	[0.640, 0.685]	
SE technology shock, σ_z	R^+	IGam[0.1,Inf]	0.321	[0.306, 0.334]	
SE investment shock, σ_a	R^+	IGam[0.1,Inf]	0.564	[0.527, 0.600]	
SE preference shock, σ_Δ	R^+	IGam[0.1,Inf]	0.467	[0.445, 0.488]	
SE government shock, σ_g	R^+	IGam[0.1,Inf]	0.943	[0.894, 0.992]	
SE collateral shocks, σ_{θ}	R^+	IGam[0.1,Inf]	0.145	[0.133, 0.156]	
SE measurement error, σ_y^{me}	[0, 0.29]	Uniform	0.290	[0.289, 0.290]	
SE measurement error, σ_s^{me}	[0,27.42]	Uniform	27.29	[27.12, 27.42]	
Technology shock effect, Ω_z	[-3,3]	Uniform	1.054	[0.924, 1.187]	
Investment shock effect, Ω_a	[-3,3]	Uniform	0.277	[0.188, 0.371]	
Preference shock effect, Ω_{Δ}	[-3,3]	Uniform	0.729	[0.644, 0.818]	
Government shock effect, Ω_g	[-3,3]	Uniform	0.255	[0.203, 0.305]	
Collateral shock effect, Ω_{θ}	[-3,3]	Uniform	1.546	[1.186, 1.931]	

Table 3.13: Estimation (transitory TFP)

Figure 3.8 and 3.9 show the estimated model's total factor productivity series compared with Fernald's (2014) total productivity series, as well as the index of

estimated confidence compared with the U.S. Business Confidence index for the estimation with transitory technology shock.



Figure 3.8: Fernald's vs model's total factor productivity (annual data).



Figure 3.9: Business confidence index vs animal spirits shocks (normalized data).

Figure 3.10 and 3.11 show the estimated model's total factor productivity series compared with Fernald's (2014) total productivity series, as well as the index of estimated confidence compared with the U.S. Business Confidence index for the estimation with MEI shock.



Figure 3.10: Fernald's vs model's total factor productivity (annual data).

3.A.2 Wedges

Business cycle accounting has been introduced by Chari et al. (2007). Brinca et al.'s (2016) interpretation of the Great Recession in terms of both the labor as well as financial wedges (denoted by τ_t^x). In terms of a benchmark prototype economy, the labor wedge is introduced via the household's period budget constraint

$$\dots = (1 - \tau_t^n) w_t N_t + r_t u_t K_t.$$

hence it is like a tax on labor services. The labor wedge $1-\tau_t^n$ is constructed from the intratemporal first-order condition that is a wedge between the marginal rate of substitution and the marginal product of labor. In log-linear form, it would write as

$$\underbrace{\left(\underline{\eta}\widehat{N}_t + \widehat{c}_t\right)}_{\mathrm{MRS}_{C,l}} - \underbrace{\left(\widehat{y}_t - \widehat{N}_t\right)}_{\mathrm{MPL}} = \frac{\tau^n}{\tau^n - 1}\widehat{\tau}_t^n.$$



Figure 3.11: Business confidence index vs animal spirits shocks (normalized data).

The model's labor wedge is driven by fluctuations of both the markup as well as stochastic preferences. Chari et al. (2007) introduce in their business cycle accounting framework an investment wedge to measure distortions that would occur capital and financial markets. It is like a tax on investment. As the relative price (that we use as observable) maps exactly into this wedge in our artificial economy, we decided to turn to a slightly different measure of capital market distortions as do Kobayashi and Inaba (2006) as well as Cavalcanti et al. (2008).¹⁹ The capital wedge τ_t^k is introduced via the household's period budget constraint

$$\dots = w_t N_t + (1 - \tau_t^k) r_t u_t K_t$$

 $^{^{19}\}mathrm{In}$ fact, Kobayashi and Inaba (2006) prove an equivalence of the capital wedge as well as the investment wedge.

Hence it is like a tax on capital services. This then implies from capital utilization's first order condition that

$$1 - \tau_t^k = \frac{\delta_0}{\alpha} A_t u_t^{1+\nu} K_t / Y_t$$

which allows to compute the empirical wedge from available data of the right hand side variables (rather than using the intertemporal Euler equation). In terms of our original model, the capital wedge equals the inverse of the markup. In a log-linearized world, we have a relation of the artificial wedge $\hat{\tau}_t^{m,k}$ and marginal costs $\hat{\phi}_t$ as

$$\widehat{\tau}_t^{m,k} = -\frac{1-\tau^{m,k}}{\tau^{m,k}}\widehat{\phi}_t.$$

In the steady state, $1 - \tau^{m,k}$ equals ϕ which, of course, is the inverse of the markup. Given data on the relative price, utilization rates, output and capital constructed using

$$K_{t+1} = \left(1 - \delta_0 \frac{u_t^{1+\nu}}{1+\nu}\right) K_t + I_t$$

as well as a parameter calibration, one can compute an empirical series for the capital wedge. We then use the estimated model and the implied series for $\hat{\tau}_t^{m,k}$ to construct a series of the model-wedge $\tau_t^{m,k}$. The model wedge replicates the overall empirical pattern as well as the depth of the distortions associated with the market of capital. The investment wedge in Figure 3.7 is computed from the original Chari et al. (2007) formulation, that is the wedge shows up as

$$\frac{1}{1+\widetilde{\tau}_t^x}$$

From this we construct a series for $1 - \tau_t^x \equiv (1 + \tilde{\tau}_t^x)^{-1}$ and report the realizations for τ_t^x in Figure 3.7. While, by construction, not identical, the two series $-\{\tau_t^k\}$

and $\{\tau_t^x\}$ – are very similar.

3.A.3 Bianchi and Nicolò (2017)

We briefly set out the methodology that we apply in Section 3.6. It closely follows Bianchi and Nicolò (2017) and it does not require to know the (analytical solution) of the boundaries of the determinacy region.²⁰ The parameters of the log-linearized benchmark model are contained in the vector

$$\Theta \equiv [\alpha, \phi, \mu^y, \mu^a, \mu^k, \delta, \nu, \eta, \beta, \gamma, G/Y, \rho_x, \rho_a, \rho_\Delta, \rho_g, \rho_\theta, \sigma_x, \sigma_a, \sigma_\Delta, \sigma_g, \sigma_\theta].$$

The linear rational expectations (LRE) model can be rewritten in the canonical form

$$\Gamma_0(\Theta)s_t = \Gamma_1(\Theta)s_{t-1} + \Psi(\Theta)\varepsilon_t + \Pi(\Theta)\eta_t, \qquad (3.3)$$

where

$$s_{t} = [\hat{y}_{t}, \hat{c}_{t}, \hat{\imath}_{t}, \hat{N}_{t}, \hat{k}_{t+1}, \hat{u}_{t}, \hat{\phi}_{t}, E_{t}(\hat{y}_{t+1}), E_{t}(\hat{c}_{t+1}), E_{t}(\hat{\phi}_{t+1}), E_{t}(\hat{u}_{t+1}), \hat{a}_{t}^{g}, \hat{\mu}_{t}^{y}, \hat{\mu}_{t}^{k}, \hat{\mu}_{t}^{x}, \hat{\mu}_{t}^{a}, \hat{\Delta}, \hat{g}_{t}, \hat{\theta}_{t}]'$$

is a vector of endogenous variables, $\varepsilon_t = [\varepsilon_t^x, \varepsilon_t^a, \varepsilon_t^\Delta, \varepsilon_t^g, \varepsilon_t^\theta]'$ is a vector of exogenous shocks, and $\eta_t = [\eta_t^y, \eta_t^c, \eta_t^\phi, \eta_t^u]'$ collects the one-step ahead forecast errors for the expectational variables of the system. Since our model can generate at most one degree of indeterminacy, Bianchi and Nicolò suggest to append the original linear rational expectations model (3.3) with the autoregressive process

$$\omega_t = \varphi^* \omega_{t-1} + \upsilon_t - \eta_{f,t} \tag{3.4}$$

 $^{^{20}}$ Bianchi and Nicolò (2017) show that their characterization of indeterminate equilibria is equivalent to Lubik and Schorfheide (2003).

where v_t is the sunspot shock and $\eta_{f,t}$ can be any element of the forecast errors vector η_t . We choose $\eta_{f,t} = \eta_t^y$. The variable φ^* belongs to the interval (-1,1) when the model is determinate or it is outside the unit circle under indeterminacy. Under determinacy the Blanchard-Kahn condition is satisfied and the absolute value of φ^* is inside the unit circle since the number of explosive roots of the original LRE model in (3.3) already equals the number of expectational variables in the model. Then the autoregressive process ω_t does not affect the solution for the endogenous variables s_t . On the other hand, under indeterminacy the Blanchard-Kahn condition is not satisfied. The system is characterized by one degree of indeterminacy and it is necessary to introduce another explosive root to fulfill the Blanchard-Kahn condition – the absolute value of φ^* falls outside the unit circle. Denoting the newly-defined vector of endogenous variables $\hat{s}_t \equiv (s_t, \omega_t)'$ and the vector of exogenous shocks $\hat{\varepsilon}_t \equiv (\varepsilon_t, v_t)'$, then the system (3.3) and (3.4) can be condensed into

$$\hat{\Gamma}_0 \hat{s}_t = \hat{\Gamma}_1 \hat{s}_{t-1} + \hat{\Psi} \hat{\varepsilon}_t + \hat{\Pi} \eta_t$$

where

$$\hat{\Gamma}_0 \equiv \begin{bmatrix} \Gamma_0(\Theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \quad \hat{\Gamma}_1 \equiv \begin{bmatrix} \Gamma_1(\Theta) & \mathbf{0} \\ \mathbf{0} & \varphi^* \end{bmatrix}$$

and

$$\hat{\Psi} \equiv \left[\begin{array}{cc} \Psi(\Theta) & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{array} \right], \ \hat{\Pi} \equiv \left[\begin{array}{cc} \Pi_n(\Theta) & \Pi_f(\Theta) \\ \mathbf{0} & -\mathbf{I} \end{array} \right].$$

The matrix $\Pi(\Theta)$ in (3.3) is partitioned as $\Pi(\Theta) = [\Pi_n(\Theta) \quad \Pi_f(\Theta)]$ without loss of generality. Figure 3.2 shows that the model's (in-)determinacy regions. To start with, the prior probability of determinacy or indeterminacy is set. The prior probability for determinacy, indeterminacy and source is 52:47:1 in percent. All priors are as in benchmark cases with the exception of the prior for the elasticity of the collateral constraint γ which is now beta-distributed, centered at 0.17 with standard deviation 0.1 and we truncate it to be no more than 0.61. Following Bianchi and Nicolò (2017), the determinacy model is estimated by fixing the parameter φ^* to a value smaller than one (e.g. 0.5) in a way that the model is solved only under determinacy while the indeterminacy model is estimated by fixing φ^* greater than one (e.g. 1.5) in a way that the model is solved only under indeterminacy. All parameters that pertain to the solution under indeterminacy are restricted to zero when we estimate the determinacy model. Lastly, we report the estimation results for the two versions of the model. The "Indeterminacy" column shows that using the alternative estimation method has only a very small effect on the paper's main results in regards to parameter estimates.

3.A.4 Determinacy versus indeterminacy

Table 3.14 shows, the estimated parameters that arise from applying Bianchi and Nicolò (2017) are essentially equivalent to our previous results (e.g. Table 3.2) and thus the implications regarding the important role of animal spirits persist.

Table 3.15 shows the variance decomposition for the determinacy model with technology and MEI shocks.

		Determinacy		Ind	eterminacy
Parameters	Density[mean, std]	Mean	90% Interval	Mean	90% Interval
ϕ	Beta[0.88, 0.01]	0.891	[0.884, 0.899]	0.833	[0.831, 0.834]
γ	Beta[0.17, 0.10]	0.001	[0.000, 0.002]	0.322	[0.315, 0.329]
ψ_{yq}	Beta[0.5, 0.2]	0.997	[0.996, 0.998]	0.965	[0.953, 0.977]
$\stackrel{\circ}{x}$	IGam[44,Inf]	10.48	[9.57, 11.34]	47.37	[44.24, 50.43]
$ ho_x$	Beta[0.5, 0.2]	0.042	[0.031, 0.053]	0.025	[0.008, 0.042]
$ ho_a$	Beta[0.5, 0.2]	0.083	[0.073, 0.092]	0.029	[0.013, 0.045]
$ ho_\Delta$	Beta[0.5, 0.2]	0.961	[0.955, 0.966]	0.984	[0.981, 0.988]
$ ho_q$	Beta[0.5, 0.2]	0.935	[0.923, 0.946]	0.986	[0.982, 0.989]
$\rho_{ heta}$	Beta[0.5, 0.2]	0.982	[0.978, 0.985]	0.992	[0.990, 0.994]
σ_η	IGam[0.1, Inf]			0.862	[0.823, 0.904]
σ_x	IGam[0.1, Inf]	0.546	[0.520, 0.572]	0.690	[0.645, 0.733]
σ_a	IGam[0.1,Inf]	0.544	[0.510, 0.578]	0.562	[0.525, 0.598]
σ_{Δ}	IGam[0.1, Inf]	0.608	[0.582, 0.633]	0.386	[0.364, 0.407]
σ_g	IGam[0.1, Inf]	1.106	[1.049, 1.166]	0.945	[0.896, 0.993]
$\sigma_{ heta}$	IGam[0.1, Inf]	0.258	[0.245, 0.270]	0.132	[0.121, 0.143]
σ_{y}^{me}	Uniform	0.290	[0.289, 0.290]	0.290	[0.289, 0.290]
σ_s^{me}	Uniform	27.40	[27.37, 27.42]	27.28	[27.10, 27.42]
$\rho(x, \eta^y)$	Uniform			-0.406	[-0.466, -0.347]
$\rho(a, \eta^y)$	Uniform			0.173	[0.112, 0.234]
$\rho(\Delta, \eta^y)$	Uniform			0.387	[0.336, 0.437]
$ ho(g,\eta^y)$	Uniform			0.275	[0.225, 0.326]
$ ho(heta,\eta^y)$	Uniform			0.151	[0.090, 0.212]

Table 3.14: Estimation (Determinacy vs Indeterminacy)

Table 3.15: Unconditional variance decomposition (Determinacy, MEI shock)

Series/shocks	ε_t^x	ε^a_t	ε^{Δ}_t	ε^g_t	ε_t^{MEI}	$\varepsilon_{y,t}^{me}$	$\varepsilon_{s,t}^{me}$
$\ln{(\boldsymbol{Y}_t/\boldsymbol{Y}_{t-1})}$	25.93	11.24	16.37	10.60	30.49	5.36	0.00
$\ln{(\boldsymbol{C}_t/\boldsymbol{C}_{t-1})}$	44.73	2.73	49.00	0.99	2.56	0.00	0.00
$\ln\left(A_t I_t / A_{t-1} I_{t-1}\right)$	18.65	17.13	5.87	6.08	52.28	0.00	0.00
$\ln{(N_t/\bar{N})}$	2.57	3.79	7.51	13.40	72.73	0.00	0.00
$\ln{(G_t/G_{t-1})}$	19.39	3.53	0.00	77.08	0.00	0.00	0.00
$\ln{(A_t/A_{t-1})}$	0.00	100	0.00	0.00	0.00	0.00	0.00
Credit spread	0.00	0.00	0.00	0.00	93.87	0.00	6.13

3.A.5 Data description

This appendix is to describe the details of the source and construction of the data used in estimation. The sample period covers the first quarter of 1955 through the fourth quarter of 2014:

 Real Gross Domestic Product. Billions of Chained 2009 Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.6.

 Gross Domestic Product. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

3. Personal Consumption Expenditures, Nondurable Goods. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

 Personal Consumption Expenditures, Services. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

5. Gross Private Domestic Investment, Fixed Investment, Residential. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

Gross Private Domestic Investment, Fixed Investment, Nonresidential.
 Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 1.1.5.

 Government Consumption Expenditure. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 3.9.5.

8. Government Gross Investment. Billions of Dollars, Seasonally Adjusted Annual Rate. Source: Bureau of Economic Analysis, NIPA Table 3.9.5. Nonfarm Business Hours. Index 2009=100, Seasonally Adjusted. Source: Bureau of Labor Statistics, Series Id: PRS85006033.

Relative Price of Investment Goods. Index 2009=1, Seasonally Adjusted.
 Source: Federal Reserve Economic Data, Series Id: PIRIC.

Civilian Noninstitutional Population. 16 years and over, thousands.
 Source: Bureau of Labor Statistics, Series Id: LNU00000000Q.

12. Confidence: Business Tendency Survey for Manufacturing, Composite Indicators, OECD Indicator for the United States, Series Id: BSCICP03USM665S.

13. Total Factor Productivity. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity", retrieved from

http://www.frbsf.org/economicresearch/economists/john-fernald/.

14. Moody's Seasoned Baa Corporate Bond Yield, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

15. Moody's Seasoned Aaa Corporate Bond Yield, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

16. 10 Year Treasury Constant Maturity Rate, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

17. Effective Federal Funds Rate, Not Seasonally Adjusted, Average of Daily Data, Percent. Source: Board of Governors of the Federal Reserve System.

18. Capacity Utilization: Total Industry (TCU), Percent of Capacity, Seasonally Adjusted, Source: Board of Governors of the Federal Reserve System.

19. GDP deflator = (2)/(1).

20. Real Per Capita Output, $Y_t = (1)/(11)$.

- 21. Real Per Capita Consumption, $C_t = [(3) + (4)]/(19)/(11)$.
- 22. Real Per Capita Investment, $I_t = [(5) + (6)]/(19)/(11)$.
- 23. Real Per Capita Government Expenditure, $G_t = [(7) + (8)]/(19)/(11)$.
- 24. Per Capita Hours Worked, $N_t = (9)/(11)$.
- 25. Credit spread = (14) (16).

Chapter 4

Do Animal Spirits Models Really Exhibit Business Cycle Behavior?

4.1 Introduction

The empirical work on the modern business cycle theory is usually linked to the pioneering studies of Arthur F. Burns and Wesley C. Mitchell who provide the concept of the business cycle and introduce a system of descriptive statistics to capture key business cycle features. Their methods of business cycle analysis which is articulated in the book *Measuring Business Cycles* (1946), has played a central role in the historical development of theoretical and quantitative descriptions of business cycle behavior. Burns and Mitchell introduce statistical tools to analyze a various of macro- as well as micro-economic time series, and their classical definition of the business cycles continue to be used to this day. In particular, their work forms the basis of the work by the NBER to help compile an official business cycle chronology for the United States.

Insights provided by Burns-Mitchell methods of business cycle analysis has guided the development of economic models and given a standard to evaluate theoretical models. The most well-known example is the so-called Adelman test, which Adelman and Adelman (1959) analyzed the early Keynesian models of Klein and Goldberger (1959). They were interested in the question if the time series generated by a stochastically perturbed variant of the Klein and Goldberger model can replicate the stylized facts of business cycles using Burns and Mitchell's methodology. Their findings have proved that the Klein-Goldberger model is quantitatively successful and pass the test. Similar macroeconomic model tests can be found in King and Plosser (1994) and Simkins (1994) in which they constructed Adelman tests for a representative real business cycle model that is driven by the stochastic process for total factor productivity.

The question asked in this paper is whether or not the animal spirits models would meet the standards laid down by the Adelman's. The motivation for this study originates from two observations in the previous literature. One is that previous work on model assessment has focused on the unique equilibrium model such as real business cycle model. However, multiple equilibrium models that are driven by i.i.d. animal spirits shocks have not been subjected to scrutiny by Burns and Mitchell methodology; the other one is that the early studies use artificially simulated shocks as model driving forces. Instead, the paper uses a sequence of estimated belief shock data. The common strategy used to assess a model is to compare the second-moment behavior of simulated series and actual economy. Here, I employ the methods of business cycle analysis developed by Burns and Mitchell to appraise the capacity of the artificial animal spirits economy in terms of whether the model can reproduce qualitative features of U.S. business cycle experience.

The paper evolves as follows. Section 4.2 provides a brief overview of Burns-Mitchell approach. This methodology is applied to the growth cycle of the post-war U.S. data in section 4.3. Section 4.4 provides an overview of a multiple equilibrium models driven by animal spirits shocks with stochastic growth. In this section, I also compare the business cycle behavior of U.S. and model generated time series using the methods developed by Burns and Mitchell. Section 4.5 makes a crude comparison between the stochastic properties of business cycle analysis such as standard deviation and correlation and the summary measure proposed by Burns and Mitchell. Section 4.6 concludes.

4.2 Burns-Mitchell methodology

The Burns and Mitchell's methodological approach is completely well documented in their 1946 treatise, *Measuring Business Cycles* and a concise summary of it as well as its implementation in a general equilibrium context can be found in the published papers of King and Plosser (1994) and Simkins (1994). I therefore briefly describe main ideas of the Burns and Mitchell method and my implementation of their procedures.

Burns and Mitchell investigated the central characteristics of the business cycle of economic time series by constructing reference-cycle patterns. In their analysis, these reference-cycle patterns are the necessary tool used to examine the cyclical behavior of different economic time series. The first step in constructing these reference-cycle patterns is to determine the periods of expansion and contraction or, namely, to specify the turning points (peaks and troughs) in the reference cycle. The business cycle series consists of a sequence of reference cycles, measured trough-to-trough by convention.

With the reference cycle defined by the turning point, each complete reference cycle is divided into nine stages (I to IX). For quarterly data, I simply set each month of the quarter equal to the quarterly value and proceeding to treat the series as monthly. Stage I centered on the initial trough; Stage V centered on the reference peak, and stage IX centered on the terminal trough. The expansion phase (stages I to V) is divided into three substages (II, III, and IV) of equal length (excluding months contained in stages I and V). The contraction phase (stages V to IX) is measured in an analogous fashion. Next, each observation in the cycle is expressed as a percentage of the cycle base called cycle relatives, where the cycle base is the mean monthly value assumed between stage I and IX. Thus a value of 100 is attributed to the average value of the series over the cycle; all other values are compared with 100. This procedure removes the inter-cycle trend while leaves the intra-cycle trend unaltered. Last, mean cycle relatives per stage are averaged across all reference cycles to yield a graphical summary of an average business cycle in the nine-point-plot. Besides the visually appraising the key business cycle characteristics of economic time series, the nine-point graph also provide descriptive statistics about (1) the lead-lag relationship between individual time series and aggregate economic activity, (2) the volatility of a series over the business cycle (i.e. amplitudes), and (3) the coherence of a time series with business cycle fluctuations (i.e. conformity).

(1) Lead-Lag relationship. For individual series that is closely related to business cycle movement, the maximum value occurs in stage V (peak) in nine-point graphs. This is based on the business cycle measure through-to-through. Departure from this pattern indicates that the individual time series has a particular lead-lag relationship with the aggregate economic activity. For instance, if a series peaks in stage IV rather than in stage V, this implies that this series typically lead at business cycle peaks.

(2) Amplitudes. The amplitude summarizes the volatility of the series over the business cycle and is measured for both expansions and contractions as well as the full cycle. The amplitude over the expansion phase is simply calculated as the cycle relative at the peak (stage V) minus the initial trough (stage I) cycle relative, while the amplitude over the contraction phase is calculated as the cycle relative at the terminal trough (stage IX) minus the stage V cycle relative. The amplitude will be positive for expansion and negative for contractions if a series typically rise during expansions and falls during contractions. The amplitude over the business cycle is simply the difference between the expansion and contraction amplitudes.

(3) Conformity. Burns and Mitchell use the indexes of conformity to quantitatively measure the degree to which a specific time series coincides with the business cycle. Indexes of conformity are computed for expansions, contraction, and the full cycle. A series' expansion conformity index is calculated by assigning a value of +100 to each expansion for which the average per month change in the cycle relative from stage I to stage V is positive and assigning a value of -100 for each cycle in which the average per month change is negative (the series falls during an expansion). Taking the average of these values over all reference cycles gives the conformity index. Basically, an expansion index value near +100 indicates that a variable moves procyclical in the expansion phase and -100 if it moves anticyclically with business cycle expansions. Values close to zero implies that the series lacks the cyclical correlation with business cycle expansions. The contraction conformity index is measured analogously (e.g. a value of +100 is assigned if the average per month change in the cycle relative from stage V to

stage IX is negative). The full cycle conformity is determined by the relative behavior of the series during both expansion and contraction phases. A value of +100 is assigned if the rate of change in the cycle relatives during the expansion phase is greater than the rate of change in the cycle relatives during a reference cycle contraction and -100 if not. Therefore, a series may continue rising during a contraction, that is, conformity is +100 during expansions and -100 during contractions, but if the rate of monthly increase in contractions is less quickly than during an expansion, the full cycle conformity would be +100.

4.3 U.S. business cycle behavior

In this section, I employ the Burns and Mitchell's methodology described above to characterize the cyclical fluctuations of a set of post-war U.S. macroeconomic data. The main focus is on four commonly discussed macroeconomic time series: real GDP, consumption of nondurables and services, gross fixed investment, nonfarm business hours worked. These data are quarterly, seasonally adjusted, per capita and cover the period 1955.2-2014.4. Following Simkins (1994), I focus on the cyclical behavior of growth cycle (trend-adjusted cycle) rather than on *classical business cycle* (absolute value) of series which developed by Burns and Mitchell. To characterize fluctuations in economic activity in a growing economy, in which there may be long periods in which no classical business cycle are observed (e.g. there was no classical contraction in the U.S. between February 1961 and December 1969, a period of 106 months) or there may be insufficient number of classical business cycles (e.g. there was no conventional business cycle in the West German economy between 1950 and 1966), Mintz (1969) suggested to concentrate on growth cycle instead. Hence, I will keep the analysis in the same framework developed by Burns and Mitchell

by simply removing the long-run trends from data. For the simulated model, the generated series are represented as percentage deviations from the long run growth path, suggesting the growth cycle framework. Thus an Adelman-test for growth cycles can be constructed to evaluate the performance of the animal spirits models. I use the Hodrick and Prescott filter to eliminate the stochastic trend from all U.S. time series.

I use OECD 'growth cycle' reference turning points for United States to determine the peak and troughs of the reference cycle for both U.S. and artificially generated data. Table 4.1 sets out both the 'classical' business cycle turning points selected by the NBER and the 'growth' cycle chronologies constructed by the OECD. It also presents the duration (in months) of the classical and growth cycles. The classical cycles must be no less than 15 months long and both expansions and contractions phases must have a duration of at least six months. The completed growth cycles must be at least 18 months in duration, and growth cycle phases must last at least nine months. As can be seen from the table, there are clearly more turning points in the growth cycle than in the classical cycle, with eight completed (trough-peak-trough) classical cycles and twelve growth cycles over our sample period. In general, peaks in growth cycles tend to lead those observed by the classical approach. Some of these growth cycles merge into the well-known recessions of aggregate economic activity, while others do not. The U.S. classical cycle is asymmetric—average contraction (12 months) is considerably shorter in duration than expansion (65 months). By contrast, the U.S. growth cycle is much more symmetrical in expansion and contraction phase durations. Typical contraction phases last about 22 months, while typical expansion phases are slightly longer at about 33 months.

	nths	on Cycle ^a		34	25	54	38	53	91	52	53	52	86	75	47		55
	tion in mor	Contractio	31	20	11	18	20	23	48	18	26	15	33	19	12		22
rowth Cycle	Dura	Expansion		14	14	36	18	30	43	34	27	37	53	56	35		33
IJ	rning points	Peak	1955.9	1959.6	1962.4	1966.3	1969.3	1973.5	1978.11	1985.9	1989.6	1994.9	2000.5	2007.10	2012.4		Avg.
	OECD tu	Trough		1958.4	1961.2	1963.3	1967.9	1970.11	1975.4	1982.11	1987.3	1991.8	1995.12	2003.2	2009.5	2013.4	
		$Cycle^{a}$		34	117	52	64	28	100	128	91						77
	tion in months	Contraction	×	10	111	16	9	16	×	∞	18						12
assical Cycle	Durat	Expansion		24	106	36	58	12	92	120	73						65
CI	rning points	Peak	1957.8	1960.4	1969.12	1973.11	1980.1	1981.7	1990.7	2001.3	2007.12						Avg.
	NBER tui	Trough		1958.4	1961.2	1970.11	1975.3	1980.7	1982.11	1991.3	2001.11	2009.6					

Table 4.1: Turning points in the post-war period (1955-2014)

The growth cycle turning point chronologies shown in Table 4.1 are used to construct Burns and Mitchell cyclical measures for postwar U.S. economy. Figure 4.1 displays the average behavior, in cycle relatives, over the nine stages of the business cycle for each of the four macroeconomic time series. Each of these figures is constructed in the same manner and drawn to the same scale. The plot shows the average cycle relative for each stage where the average is taken over the 12 complete trough-peak-trough cycles beginning with the trough in 1958:II and ending with the trough in 2013:II. Stage I coincides with the initial trough, stage V with the peak, and stage IX corresponds to the terminal trough.





Figure 4.1: Average behavior over 12 business cycle stages for 4 macroeconomic time series

The four series all exhibit a distinct pro-cyclical pattern, rising during business cycle expansions and falling during contractions and they show little evidence of leads or lags at business cycle peaks or troughs. Consumption series is much smoother, and the investment displays a greater amplitude. The investment tends to rise (decline) more sharply during expansion (contraction) phases than other series do across the business cycle. Table 4.2 presents the key characteristics of the business cycle for all four U.S. macroeconomic series. These summary measures verify what is observed in the figure and provide additional information about the cyclical behavior of the aggregates. The first group of statistics measures the amplitude of each series over the business cycle. Looking at the full cycle amplitudes, the well-known business cycle facts can be observed. That is, the investment is the most volatile sector; consumption fluctuates less than output does and its amplitude is approximately one-half that of output. Hours worked is roughly as volatile as output. Conformity indexes for the U.S. series are also summarized in Table 4.2. The output, consumption, investment and hours worked series all exhibit +100 coherence, indicating that they are strongly procyclical series.

	Cycl	ical amplitude		Cyclical conformity ^a			
Series	Expansions	Contractions	Cycle	Expansions	Contractions	Cycle	
Output	4.1	-3.8	7.9	+100	+100	+100	
Consumption	1.6	-1.6	3.2	+100	+100	+100	
Investment	11.1	-10.1	21.2	+100	+100	+100	
Hours	4.0	-3.6	7.6	+100	+83	+100	

Table 4.2: Burns and Mitchell business cycle measures (U.S. data)

^a A conformity index value near +100 says that cyclical movements in the data are closely correlated with business cycle activity, while lower values indicate a lower degree of correlation with business cycle movements.

This section has used the Burns and Mitchell techniques to investigate the post-war U.S. economy. Analogous measures will be conducted in the following section for the artificial economy in the spirit of Adelman.

4.4 Cyclical behavior of an animal spirits model

There is a long tradition that attempts to explain business cycle by assigning purely extrinsic psychological factors a central role. In contrast to the standard framework with a unique equilibrium, multiple-equilibria macroeconomic models suggest that agent's self-fulfilling beliefs or animal spirits can be an important independent factor for business cycles fluctuations. One of the most influential quantitative support for animal spirits model was presented by Farmer and Guo (1994). Their work shows explicitly that a sunspots model, driven by i.i.d. animal spirits shock was capable of generating time series that mimic real-world phenomena. The volatility of simulated economy as well as contemporaneous covariances between output and other variables in the model appears to match up well with broad features of the dynamic responses in the data. In the following section, I will evaluate a particular animal spirits model in terms of whether it mimics the business cycle characteristics of post-war U.S. data when viewed through the eyes of Burns and Mitchell.

4.4.1 An animal spirits model

The artificial economy that we study here is a discrete-time adaptation of Benhabib and Wang (2013). The model features credit frictions in the form of endogenous borrowing constraints in a model of monopolistic competition in which, as usual, perfectly competitive firms produce final output by combining a continuum of differentiated intermediate inputs. Intermediate goods-producing firms are collateral-constrained in how much they can borrow to finance their working capital needs. The mechanism generating a self-fulfilling explanation stands on an endogenous and countercyclical markup channel. The economy is perturbed by a multitude of shocks, namely, permanent technology shocks, investment-specific technology shocks, preference shocks, government spending shocks, collateral shocks as well as animal spirits shocks. The model is estimated by standard Bayesian methods using U.S. data at the quarterly frequency, from 1955:I to 2014:IV. The full model economy and estimation strategy have explored in detail in Chapter 3 so that the presentation is deliberately brief.

A competitive equilibrium in this model is characterized by the following necessary conditions:

$$Y_t = (u_t K_t)^{\alpha} (X_t N_t)^{1-\alpha}$$

$$\tag{4.1}$$

$$Y_t = C_t + I_t + G_t \tag{4.2}$$

$$N_t^{1+\eta}(C_t - \Lambda_t) = \frac{1-\alpha}{\varphi} \phi_t Y_t \tag{4.3}$$

$$A_t \delta_0 u_t^{1+\nu} = \alpha \phi_t \frac{Y_t}{K_t} \tag{4.4}$$

$$K_{t+1} = (1 - \delta_t)K_t + I_t \tag{4.5}$$

$$\frac{A_t}{C_t - \Lambda_t} = \beta E_t \left[\frac{A_{t+1}}{C_{t+1} - \Lambda_{t+1}} \left(\delta_0 u_{t+1}^{1+\nu} + 1 - \delta_{t+1} \right) \right]$$
(4.6)

$$\phi_t = \tau \theta_t (\frac{Y_t}{\bar{Y}_t})^{\gamma} \tag{4.7}$$

In the above equations, the first expression is a constant returns to scale Cobb-Douglas production function with capital share parameter $\alpha \in (0, 1)$. X_t denotes the exogenous labor-augmenting technological progress. Equation (4.2) depicts the economy-wide resource constraint. The term A_t represents a nonstationary investment-specific technology shock which affects the transformation of consumption goods into investment goods. Equation (4.3) shows the intratemporal labour-consumption trade-off, where $\eta \geq 0$ represents the elasticity of labour supply, and the parameter φ denotes the disutility of working. The term Λ_t represents preference shocks and ϕ_t stands for monopolistic firms' marginal cost. Equation (4.4) determines the efficient level of capacity utilization u_t and $\nu > 0$ measures the elasticity of the depreciation rate with respect to capacity used. Law of motion for the aggregate capital stock is expressed in equation (4.5) and equation (4.6) is the consumption Euler equation. The last expression is the binding collateral constraint with the parameter restrictions $0 < \tau < 1$ and $\gamma > 0$. θ_t represents the exogenous collateral shocks which affect financial health.

 Table 4.3: Parameter values

Subjective discount factor	$\delta = 0.99$
Capital share	$\alpha = 1/3$
Labor supply elasticity parameter	$\eta = 0$
Steady-state depreciation rate	$\delta = 0.0333$
Steady-state capacity utilization rate	u = 1
Elasticity of depreciation rate parameter	$\nu = 0.5824$
Steady-state marginal cost	$\phi=0.8327$
Elasticity of collateral parameter	$\gamma=0.3216$
Steady-state government expenditure share of GDP	G/Y = 0.21
Steady-state gross per capita GDP growth rate	$\mu^{y} = 1.0041$
Steady-state gross growth rate of price of investment	$\mu^a=0.9949$

The model is then detrended and log-linearized around the steady state. Some parameters are calibrated while others are estimated (see Chapter 3). I list all the parameter values used in this paper in Table 4.3. After estimation, a real time series of animal spirits shocks were obtained. Since the goal of this paper is to evaluate the validity of the animal spirits model by comparing the resulting model-generated sequences of variables to true data, we are therefore working under the assumption that randomness in this economy is only due to agents' belief shocks. Then the dynamic system becomes

$$\begin{bmatrix} \hat{y}_{t+1} \\ \hat{k}_{t+1} \end{bmatrix} = J \begin{bmatrix} \hat{y}_t \\ \hat{k}_t \end{bmatrix} + R \begin{bmatrix} \hat{\varepsilon}_{t+1} \\ 0 \end{bmatrix}, \qquad (4.8)$$

where hat variables denote percentage deviations from their steady-state values. Here $\hat{\varepsilon}_{t+1}$ is our animal spirits shocks, which is by definition serially uncorrelated and mean zero. Given the behavior of output and capital, the remaining endogenous variables can be expressed as a linear combination of these two variables

$$\begin{bmatrix} \hat{c}_t \\ \hat{i}_t \\ \hat{n}_t \\ \hat{u}_t \\ \hat{\phi}_t \end{bmatrix} = Q \begin{bmatrix} \hat{y}_t \\ \hat{k}_t \end{bmatrix}.$$
(4.9)

By solving the model, coefficients in the matrices J and Q, implied by the parameters in Table 4.3, are given in Table 4.4. Using a sequence of belief shocks generated in previous chapter as the driving force, the model was dynamically solved over the sample period 1955:II to 2014:IV (All remaining variables were initially set to zero).

	Output \hat{y}_t	Capital \hat{k}_t
Output (next period) \hat{y}_{t+1}	1.0046	-0.0616
Capital stock (next period) \hat{k}_{t+1}	0.1190	0.9713
Consumption \hat{c}_t	0.2392	0.1840
Investment $\hat{\imath}_t$	3.8620	-0.4654
Labor input \hat{n}_t	1.0824	-0.1840
Capital utilization \hat{u}_t	0.8352	-0.6320
Marginal cost $\hat{\phi}_t$	0.3216	0

Table 4.4: Parameters of the log-linear system

4.4.2 Cyclical properties of the simulated data

The nine-point plot can be used as a mean of visually judging how closely the cyclical behavior of the U.S. data and simulated economies match up. A visual impression of the business cycle characteristics generated by an animal spirits model is provided in figures 4.2(a)-4.2(d). These business cycle plots using the methods described previously and had the same scale as those for the data. The simulated series are also H-P filtered. Comparing the pictures with figures 4.1(a)-4.1(d), it would be difficult to distinguish the simulated data's characteristics from those of the actual data. The figures show that the model series all move procyclically and reach the summit exactly the peak stage in the reference cycle. The similar general shape of the data and model series demonstrates that artificial series matches well postwar U.S. cycles.





Figure 4.2: Characteristics generated by the animal spirits model

I have also computed the measures of amplitude and conformity index for the different simulated series. These statistics are displayed in Table 4.5. The investment sector clearly exhibits the largest volatility over the reference cycle, while the consumption is the least variation sector of the simulate economy. Output fluctuates as volatile as labour hours. Compared to the full-cycle amplitude statistics presented in Table 4.2 for the U.S. economy, the simulated model is slightly less variable than those appear in the real economy except for the investment series, but the relative order for these aggregates is similar to that found in the data. For cyclical conformity, simulated investment and hours series all exhibit +100 conformity in an average full cycle and simulated output and consumption series are also procyclical but do not conform perfectly. These statistics indicate that the model series are well-conforming series, that is, generally rise during expansions and fall during contractions.

	Cycl	ical amplitude		Cycl	Cyclical conformity			
Series	Expansions	Contractions	Cycle	Expansions	Contractions	Cycle		
Output	3.0	-2.8	5.8	+100	+83	+83		
Consumption	0.7	-0.7	1.4	+83	+83	+83		
Investment	11.4	-10.7	22.1	+100	+67	+100		
Hours	3.2	-3.0	6.2	+100	+67	+100		

Table 4.5: Burns and Mitchell business cycle measures (simulated data)

Overall, above analysis illustrate that the model, driven only by a series of belief shocks, is capable of reproducing main features of U.S. business cycle behavior by employing Burns and Mitchell techniques.

4.5 Comparing alternative business cycle

measurements

The previous section illustrates that an animal spirits model can replicate major business cycle features found in the data using the techniques of Burns and Mitchell. Now I transform my focus from evaluating a model to a more systematic investigation of the Burns and Mitchell procedures themselves. In particular, I will make a crude comparison between the summary measure proposed by Burns and Mitchell and the more common business cycle statistics emphasized in the business cycle literature, such as standard deviations and cross-correlations.

	United States	Simulated	United States	Simulated						
(A) Cyclical variability ^a										
Series	Standard de	eviations	Amplit	Amplitude						
Output	1.51	1.61	7.9	5.8						
Consumption	0.87	0.50	3.2	1.4						
Investment	4.96	6.16	21.2	22.1						
Hours	1.88	1.72	7.6	6.2						
(B) Cyclical conformity										
Series	Correlation w	ith output	Conformity							
Output	1.00	1.00	+100	+83						
Consumption	0.81	0.86	+100	+83						
Investment	0.88	0.99	+100	+100						
Hours	0.86	0.99	+100	+100						

Table 4.6: Comparison of business cycle summary measures

^a The sample period is 1958:II to 2009:II.

The comparison results of these two procedures can be found in Table 4.6. One characteristic of Burns and Mitchell procedures is that the measures are computed without regard to whether the moments of the series exist or not. Hence, how to interpret the statistical properties of these measures is an important question. Nevertheless, one can compare these measures with common statistical measures of second moments. The standard deviations along with the corresponding amplitude measures of both simulated series and the U.S. data are reported in part A of Table 4.6. An easy way of comparing the two procedures is to look at the rank of the variability of the series. As is verified by Table 4.6, the ranking of the key series is essentially identical no matter one uses standard deviation or amplitude. Investment is the most volatile series, and hours worked roughly has the same volatility as output and consumption is less variable than output.

In part B of this table, I compare the correlation coefficient of each series with output and Burn and Mitchell's measure of conformity. Although the conformity measure is rather crude, it does not necessarily mean a poorer measure. The full cycle conformity may be +100 even though a series may rise during the contraction phase as long as per month change during the expansion exceeded the per month change during the contraction. It is apparent that correlation and conformity differ with reference to what they measure. Thus, I could not say which method dominates the other. The answer depends on the situation at hand.

4.6 Conclusion

This paper follows the path of Adelman and Adelman (1959), employing the classical business cycle methods of Burns and Mitchell (1946) to evaluate the cyclical properties of a particular animal spirits model. In particular, the paper evaluates if one could distinguish between the actual historical time series and the artificial series generated by a stochastically perturbed economic model. The upshot of this analysis is that animal spirits model are capable of reproducing many of the salient features of actual economies. Burns and Mitchell's methods of business cycle analysis were successfully applied to the early Keynesian model as well as the real business cycle model. As indicated here, these methods can be used profitably to analyze the cyclical behavior of animal spirits models as well.
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