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On the Computational Role of the Simple Cells in Early Vision

by

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ABSTRACT

The simple cells in feline and primate primary visual cortex are involved in the coding and early processing of spatiotemporal information acquired binocularly from the visual field. Each simple cell can be viewed as an approximately linear device characterised by its receptive field profile (RFP), a spatially reversed version of its spatiotemporal impulse response function.

The Gabor function model of the simple cell RFP is evaluated, and the recent controversy concerning the relevance to early vision of its achievement of the lower bound on joint spatial and spectral spread dictated by the Weyl-Heisenberg Uncertainty Principle is illuminated. In an investigation of the multi-dimensional signal processing performed by the simple cells, image processing and coding schemes which might explain the observed variety of simple cell spatial RFPs are reviewed. These schemes are classified into the categories of *filtering* and *decomposition*, according to whether the RFP is used as the kernel of a spatial filter, or as an expansion function whose coefficient is to be calculated for the visual image.

Artificial neural networks (ANNs) which find the least-squares solution to the set of linear equations posed by the image decomposition problem are critically reviewed, and a single-layered, linear recurrent ANN is proposed for this task. The linear neural activation function used by this network is then replaced by a more biologically plausible, piece-wise linear, saturating nonlinearity, and the resultant globally stable network is shown to effect the optimisation of more general (semi)definite quadratic forms subject to bound constraints on the optimisation variables. Although biologically plausible, these networks, when used as models of simple cell processing, are found to predict simple cell spatiotemporal RFPs whose spatial component differs in general from the chosen expansion functions. It is concluded that the simple cell spatial RFPs are *not* used as visual expansion functions, but rather as the kernels of (possibly position-dependent) spatial filters, as is suggested by their definition.