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Theory and Applications

Environmental controls with corrupt bureaucrats

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ABSTRACT. Environmental regulations typify a large class of activities in the public sector where government agencies are required to monitor the degree of compliance. These tasks are usually delegated to bureaucrats who, as self-interested agents, may engage in corrupt behavior. Such problems abound, particularly in developing countries, where corruption is regarded as one of the major causes of environmental degradation. This paper investigates the implications of corruption for the optimal design of environmental regulations and analyses the interaction between the prosecution rate, monitoring rate, and fines. It is shown that even if corruption can be deterred the fact that it may occur substantially impedes the ability of a regulator to control environmentally degrading activities.

1. Introduction

Government regulators confront problems resulting from the need to delegate administrative authority to bureaucrats, who act as their agents. The bureaucrats are thus endowed with discretionary powers. If they are assumed to be self interested, these delegated powers may be exploited for personal gain, rather than the purposes intended by the policy makers. Such problems abound, particularly in developing countries, where corruption has been shown to undermine government policy (Rose-Ackerman, 1977), impede economic growth (Mauro, 1995), and stifle the entry of new enterprises and technologies (Bardhan, 1997; Krueger, 1990; Manion, 1996).

In recent years corruption has intensified in a new domain—environmental regulation. The introduction and ratification of increasing numbers of global environmental agreements has compelled governments to introduce new and more stringent environmental controls. This, in turn, has expanded the sphere of activities through which corrupt administrators

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can extract bribes. Examples of such international agreements include the Rio Earth Summit and the CITES convention on trade in endangered species.[1]

The emerging literature on environmental performance indicates that corruption is one of the main sources of environmental damage in several countries. For instance, Desai (1998) in a comparative study of ten countries concludes that:

> corruption is a major culprit in environmental degradation. In many industrializing countries, petty corruption by mid and low level officials and bureaucrats both at the center and local level is widespread and endemic. Environmental regulations often are observed only in the breach. (page 300)

Similarly, in an econometric study of water pollution, Pargal, Mani, and Huq (1997) find that even when increased emissions prompt further inspections, these have no subsequent impact on total emissions. Corruption and inadequate penalties for violations are identified as the main factors contributing to non-compliance.

In a survey of environmental regulations O'Connor (1994: 94) highlights an analogous problem:

> In several countries studied here, the monitoring problem is compounded by weak enforcement. In short, when violators of standards are detected ... polluters are exempted from fines ... because of the power they wield.[2]

Despite the prevalence of these problems, the consequences of bribery on environmental policy outcomes remains one of the least-researched aspects of economic behavior. This paper extends the existing literature by analyzing a situation where environmental regulations create opportunities for corrupt behavior. We derive a rule to determine the optimum degree of regulation and show how it differs from the conventional Pigouvian solution for correcting externalities. The paper further analyzes the interaction between the level of environmental regulation, the required degree of monitoring, and penalties for corruption. It is shown that, even if corruption can be deterred, the fact that it may occur substantially impedes the ability of a regulator to control environmentally degrading activities.

[1] For instance the CITES convention calls for a complete ban on the commercial use and trade of species listed in Appendix 1. This, however, has done little to halt the illegal trade of tiger bones and organs from India to China, Taiwan, and Japan or rhinoceros horns from Africa and India to SE Asia—both of which are demanded for their presumed therapeutic value (TRAFFIC International, 1998).

[2] Heyes and Rickman (1999) and Harrington (1988) show that exempting violators from penalties may be optimal in some contexts. This is likely to be so if tolerating violations in one period or sphere of policy induces greater compliance in other periods or policy areas. Thus, O'Connor's observation may be consistent with welfare maximization. In contrast, the violations considered in this paper occur as a consequence of corruption and by assumption confer no other environmental benefits.

The study of corruption in an environmental context seems important for several reasons. Firstly, environmental issues are representative of a larger class of problems where the government delegates powers to self-interested bureaucrats. Thus, the results outlined in this paper may have wider applicability. In addition, many of the more acute problems of pollution and bio-diversity preservation are encountered in developing countries with high levels of corruption.[3] Accordingly, analyzing the interaction between environmental controls and corruption is of some relevance for environmental policy purposes. Finally, in international forums, such as the WTO, environmental issues have been a major source of contention. Thus, an understanding of factors that promote and inhibit corruption associated with environmental regulations and the limits to such regulations is of practical importance.

Much of the existing theoretical work on corruption deals with monitoring problems which arise in a hierarchical structure when a principal (such as the government) confers supervisory powers upon a self-interested agent (say an inspector). The literature explores whether bribe taking can be deterred through incentive payments and fines. The central conclusions which emerge are that: marginal increases in a fine imposed on the bribe taker leads to higher bribes being paid in equilibrium (Mookherjee and Png, 1995). In contrast, penalties imposed on the bribe giver, unambiguously reduce the level of corruption (Basu, Bhattacharya, and Mishra, 1992). Payment of a sufficiently high efficiency wage diminishes the gains from bribe taking and may under certain conditions deter corruption (Besley and McLaren, 1993).

A distinct literature on environmental regulation has also developed, which focuses upon compliance behavior and penalties (Keller, 1991; Malik, 1990; van Egteren and Weber, 1996; Heyes and Rickman, 1999). However, to our knowledge the problem of corruption has been largely ignored in the growing literature on environmental compliance.

Corruption has two non-trivial characteristics which makes the real effects of environmental regulations very different from their legal properties. First, when bureaucrats accept bribes this has the effect of diluting the sanctions for non-compliance. Since the expected penalty for non-compliance declines, it can be more difficult for the regulator to control the environmentally degrading activity. More importantly, corruption usually flourishes in situations where information can be concealed from a regulator. Thus, policies must be based on the (potentially) distorted information provided by a bureaucrat. The aim of this paper is to address these and other related issues, which to our knowledge have not been previously examined.

[3] While statistical estimates are hard to obtain because of the clandestine nature of corrupt activities, examples abound. Some striking cases include: the widely publicized illegal burning of forests in Sumatra and the consequent pallor of smog across Singapore and Malaysia (*The Economist*, 18 August 1999), forest clearance in Madagascar (World Bank, 1999), organized poaching of tigers and elephants and the buoyant illegal trade (Damania, 2001).

We consider a stylized model in which a firm emits pollution which a government regulator attempts to control through an emission tax.[4] The regulator cannot directly observe the level of pollution emitted and therefore employs an environmental inspector to monitor pollution levels. The tax paid by the firm is therefore based on the level of emissions reported by the inspector. This clearly creates an opportunity for the inspector and firm to engage in corrupt behavior by colluding and underreporting true emission levels. The regulator chooses to undertake an audit of the firm's emissions with some probability which is linked to the regulator's expectation that emissions have been underreported. With some exogenously given probability, the audit unearths true emission levels and a fine is imposed on both the firm (briber) and environmental inspector (recipient of the bribe) for underreporting discharge levels.

The analysis is based on the following sequence of events. In the first period the government sets the policy instruments (that is, the emission tax schedule, penalty for underreporting emissions, and the audit schedule). In the second stage, the firm is visited by an inspector who is assigned the task of reporting emission levels to the regulator. If the firm offers the inspector a bribe, actual and reported emissions are chosen to maximize the joint payoffs of the firm and inspector. The resulting bribe is determined by a Nash bargain. As usual, the model is solved by backward induction.

We begin by determining the equilibrium level of reported and actual emissions and examine the response of each to changes in various policy instruments. It is shown that an increase in the emission tax has the predictable effect of inducing a decline in both reported emissions and actual pollution. Reported emissions fall because a higher tax raises the cost of compliance and thus increases the payoffs from tax evasion. On the other hand, actual emissions decline because the tax increases costs so that output and emission levels fall. It is further demonstrated that a higher fine for corruption leads to an increase in reported emissions. This simply reflects the fact that the fine dilutes the expected gains from bribery. However, in some circumstances a higher fine is shown to increase actual emissions. This occurs only when the judicial system is highly inefficient, in the sense that the probability of being prosecuted lies below a threshold level. Intuitively, since a higher fine leads to an increase in reported emissions, the firm is compelled to pay more tax (on the reported emissions). When the prosecution rate is sufficiently low, the firm seeks to recover these costs by increasing actual discharges.

These results suggest that controlling emissions and the degree of compliance is likely to be a complex process, which depends upon the responsiveness of actual and reported emissions to each policy instrument. Moreover, the regulator is compelled to determine the optimal policy mix without being able to observe true emission levels. A rational regulator, who relies on reported emissions, will predict that the firm and inspector will collude and underreport emissions, if it is to their advantage. From the

[4] In order to abstract from problems of firm strategic behavior we focus on the case of a single monopolist.

revelation principle it is known that in these circumstances the regulator's optimal response can be restricted to the set of policies which induce the firm and inspector to truthfully reveal emission levels (Laffont and Tirole, 1998: 120). Accordingly, section 3 derives the welfare-maximizing response under the constraint that truthful revelation is incentive compatible.

It is demonstrated that in the presence of corruption the optimum set of policies have certain distinctive properties. Firstly, in the welfare-maximizing equilibrium the pollution tax and audit rate must be set such that the net marginal welfare gains from each are equalized. Intuitively, higher taxes induce greater corruption, which in turn necessitates greater auditing. Since auditing is costly, the optimum policy weighs the net marginal benefits from auditing against those from the emission tax. This rule thus differs from the conventional (Pigouvian) approach which requires that the emission tax be set equal to the marginal damage from pollution.

The requirement that truthful revelation is incentive compatible places further important restrictions on the tax and audit schedules. It is shown that truthtelling requires a tax schedule that rises at a decreasing rate (that is, is concave) in emissions. Intuitively, when the tax rises at a diminishing rate, this weakens the incentive to engage in corrupt behavior as emission levels increase. Truthful revelation also requires that the audit rate must decline at a decreasing rate in reported emissions (that is, be convex in reported emissions). Such an audit schedule ensures that low reports are audited more intensively, and that the audit rate does not decline too rapidly as reported emissions rise. Thus, a low polluter (who faces the more steeply rising segment of the tax schedule) is deterred from underreporting because of the higher audit rates. In contrast, a high polluter is induced to report honestly because of the gradually rising taxes which accompany the slowly declining audit rates.

A further result which emerges is that if the prosecution rate is extremely low, which occurs when the judicial system is highly inefficient, the optimum solution is to abandon attempts to regulate emissions. Intuitively, since there is no workable mechanism to enforce compliance, there is little point in expending resources on auditing emissions. If emissions are not audited there is complete non-compliance. Thus, zero pollution taxes are optimal. In policy terms low prosecutions are most likely when the judicial system is in need of institutional reform. In these circumstances environmental controls simply induce greater corruption, but do little to prevent the ensuing damage.

The remainder of the paper is organized as follows. Section 2 outlines the basic model and investigates the impact of the policy instruments on actual and reported emissions levels. Section 3 deals with the welfare-maximizing response of the regulator and section 4 concludes the paper.

2. The model

A firm which discharges pollution emissions, denoted $e \in [\underline{e}, \bar{e}]$ is visited by an environmental inspector who reports its emission levels to a regulatory agency.[5] The regulator imposes an emission tax, which is levied on

[5] The basic structure of the model is similar to that of Mookherjee and Png (1995).

reported pollution emissions of $\hat{e}$. The emission tax burden is denoted $t(\hat{e}, \tau)$, where τ is the tax rate on reported emissions. It is assumed that $\partial t/\partial \hat{e} > 0$ and $\partial t/\partial \tau > 0$, so that the tax burden is increasing in both reported emissions and the tax rate.

The firm may seek to lower its tax burden by offering the inspector a bribe of B to underreport emissions. An inspector who accepts a bribe, reports emission levels $\hat{e}$, which differs from actual emissions of e. We assume that $\hat{e} \leq e$, so that the inspector is unable to exaggerate true pollution levels. This implies that the firm can provide irrefutable evidence of emission levels to the regulatory agency, if it so chooses. The inspector receives a fixed wage of w from the regulator.[6]

The regulatory authority cannot observe actual emission levels. In keeping with the mechanism design literature (see, for example, Baron and Myreson, 1982; Laffont and Tirole, 1998) we adopt the Bayesian approach and assume that the regulator has some prior probability distribution for the unknown parameter, e, prior to receiving a report from the inspector. We let $\omega(e)$ be the density function and $\Omega(e)$ the cumulative distribution function. Then the probability that the regulator attaches to reported emissions ($\hat{e}$) being less than actual discharges (e) is given by $\rho(\hat{e}) = (1 - \Omega(\hat{e}))$.[7] Based on this expectation of emissions being underreported, the regulator initiates an audit of emission levels with some probability $\lambda = \lambda\ (\rho(\hat{e}))$. It is supposed that the audit rate (λ), is increasing in the probability that emissions are underreported. Hence, $\partial\lambda/\partial\rho > 0$, $\partial\lambda/\partial\hat{e} < 0$.[8] The probability that an audit, once initiated, successfully detects true pollution levels and leads to a prosecution is exogenously given by $\beta \in (0, 1)$. Thus β may be viewed as an indicator of the efficiency of the judicial process.[9] The probability that an audit occurs and leads to a successful prosecution is: $\sigma = \lambda\beta$.

Having specified the monitoring regime, we now describe the penalties for underreporting emissions. Typically, the penalty for a misdemeanor is linked to the extent of a crime (Shapiro, 1988). Following judicial conven-

[6] The results continue to hold if the inspector is assumed to receive some fraction (less than unity) of the tax revenue. However, the assumption of a fixed wage is simple and realistic. It more accurately reflects the (non-performance-based) mode of remuneration in the public sector in most countries.

[7] As noted by a referee, the probability of underreporting is likely to be influenced by the type and stringency of the chosen policy. This implies that the expected probability will be conditional upon the policy regime. While this is clearly a more realistic assumption, for reasons of analytical tractability we follow the existing literature and ignore this important issue

[8] Note that $\partial\lambda/\partial\hat{e} = (\partial\lambda/\partial\rho)(\partial\rho/\partial\hat{e}) = -(\partial\lambda/\partial\rho)\omega(\hat{e}) < 0$. Given the informational structure of the model, this audit rule can be shown to be optimal in the sense that it induces greater compliance than the other main auditing rules which involve either (i) a fixed probability of monitoring, or (ii) a monitoring regime which increases with reported emissions. In note 13 we show that these rules result in a solution with lower reported emissions, and greater non-compliance.

[9] As suggested by a referee, there are alternative interpretations of β. It may be viewed as a measure of the proportion of honest bureaucrats in the judiciary, or as in Klitgaard (1998) it may proxy the pervasiveness of corruption in society at large.

tion we allow for the possibility that the fines for corruption depend on the level of underreporting. Let $v = (e - \hat{e})$ denote the level of underreporting of emissions. It is assumed that an inspector found guilty of underreporting emissions is fined an amount $I(v, \theta) \geq 0$, while the firm is fined an amount $f(v, \theta) \geq 0$; where θ defines the penalty rate. As seems reasonable, the fines for corruption are increasing in the penalty rate (θ) and the level of underreporting (v) at an increasing rate.[10] That is, for $K = I(v, \theta), f(v, \theta)$ it is supposed that $\partial K/\partial v > 0$, $\partial^2 K/\partial v^2 > 0$, $\partial K/\partial \theta > 0$, $\partial^2 K/\partial \theta^2 > 0$ and $\partial^2 K/\partial v \partial \theta > 0$.

We begin by defining the gains to a firm from corruption. Given the sequential structure of the model, the level of emissions which eventuate in equilibrium will depend on expected taxes and fines. Let $e = e(t(\hat{e}), f(v,\theta), \sigma)$ denote emission levels under corrupt behavior. Profits from emission levels (e), gross of taxes, bribes and fines, are

$$G(e) = P(e)e - C(e)$$

where $P(e)$ is price of the polluting good and $C(e)$ is production costs. For simplicity we ignore pollution abatement costs which could be incorporated into the cost function without altering any of the main results. Similarly, let $e^h = e(t(e))$ denote emissions under honest behavior, then the corresponding gross profits are defined by

$$G(e^h) = P(e^h)e^h - C(e^h)$$

with $\partial P/\partial e < 0$, $\partial C/\partial e > 0$, $\partial^2 C/\partial e^2 > 0$ and $\partial^2 G/\partial e^2 < 0$.[11]

Suppose that the firm decides to bribe the inspector an amount $B > 0$ to report emissions $\hat{e} < e$. The expected gains to the firm from offering a bribe is given by

$$U^F = [G(e) - (B + t(\hat{e}) + \sigma f(v,\theta))] - [G(e^h) - t(e^h)] \tag{1}$$

where for notational brevity $t(\hat{e}) = t(\tau,\hat{e})$ and $t(e^h) = t(\tau,e^h)$ define the firm's tax burden under corruption and honest behavior respectively.

To interpret expression (1), note that the terms in the first square parenthesis represent the expected payoffs to the firm from paying a bribe. Thus, with emissions of e, profits gross of taxes, bribes and fines are $G(e)$. The remaining terms define the expected costs of a bribe. A bribe of B induces the inspector to report $\hat{e}$, so that the firm pays emission taxes $t(\hat{e})$. With probability σ a successful audit is triggered and the firm is fined $f(v,\theta)$. The terms in the second square parenthesis represent the payoffs when the firm

[10] Considering alternative penalty structures, while useful, would substantially expand the range of cases to be considered in the model. More generally, from the first-order condition in (4), it can be shown that the assumption of fines increasing in v is optimal in the sense that a non-increasing penalty schedule results in lower reported emissions.

[11] To economize on notation no superscript is used to denote emissions under corruption, while superscript h indicates emissions under honest behavior. When the distinction between corrupt and honest behavior is not required (as occurs when truthful revelation is induced) the superscript is ignored for notational brevity.

does not offer a bribe. A firm that does not offer a bribe receives gross profits of $G(e^{h})$ and pays taxes on actual emissions of $t(e^{h})$.[12]

Similarly, the gains to an inspector from accepting a bribe of B is given by

$$U^{I} = [w + B - \sigma I(v,\theta)] - w \tag{2}$$

The first term in (2) is the fixed salary (w) received by the inspector. An inspector who chooses to accept a bribe, receives an amount B, which induces a report of $\hat{e}$. With probability σ a successful audit is initiated and leads to a fine $I(v,\theta)$ being imposed. The last term represents the payoffs from honest behavior: an inspector who does not accept a bribe simply receives a fixed salary of w.

By backward induction we begin by solving for the level of reported and actual emissions. Given a tax rate, the firm and inspector will choose reported and actual pollution to maximize the joint expected payoffs from a bribe of B. Specifically

$$\operatorname*{Max}_{\hat{e},e} J \equiv (U^{f} + U^{I}) \tag{3}$$

Solving the associated first-order conditions yields

$$\frac{\partial J}{\partial \hat{e}} = -\frac{\partial t(\hat{e})}{\partial \hat{e}} + \Gamma = 0 \tag{4}$$

$$\frac{\partial J}{\partial e} = -\frac{\partial G}{\partial e} - \sigma \frac{\partial F}{\partial v} = 0 \tag{5}$$

where $\Gamma = -\beta F \dfrac{\partial \lambda}{\partial \hat{e}} + \sigma \dfrac{\partial F}{\partial v}$, $F = I(v,\theta) + f(v,\theta)$.

Thus, equation (4) suggests that the equilibrium report satisfies the condition that the marginal tax cost of increasing reported emissions (that is, $\partial t(\hat{e})/\partial \hat{e}$), is equated to the marginal expected cost of being prosecuted (that is, Γ). In contrast, by equation (5) emissions are determined by equating the marginal benefits from production (that is $\partial G(e)/\partial e$), to the expected marginal cost of a fine from higher emissions (that is $\sigma\ \partial F/\partial v$). Observe that while the expected penalty for underreporting has a direct

[12] For simplicity we ignore the possibility of corruption further up the hierarchy (for example at the prosecution stage). The issue of corruption in hierarchies has been explored in detail by Basu, Bhattacharya, and Mishra (1992). Hierarchical corruption could be incorporated into the model as follows: let α_i be the proportion of honest bureaucrats at stage $i = 1, 2, \ldots$ in the monitoring hierarchy. For $i > 1$, the inspector in stage $(i-1)$ may bribe the stage i bureaucrat who monitors her. In a two-level hierarchy the expected payoff to the firm from offering a bribe is then $(1 - \alpha_1)[G(e) - (B + t(\hat{e}) + \lambda\alpha_2 f(v,\theta))]$. As shown by Basu Bhattacharya, and Mishra (1992) and Sanyal (2000) this alters the equilibrium parameters over which bribery occurs, but does not change the qualitative properties captured in the simpler model of equation (1). We therefore ignore the complications of hierarchical corruption hereafter.

effect on actual emissions, the tax on pollution (τ) only has an indirect impact through its effect on the expected fine.[13]

Once reported emission levels have been decided, the equilibrium bribe is determined by a Nash bargain between the firm and each inspector. Each party is assumed to have equal bargaining power and the bribe is chosen to maximize the following Nash bargain

$$\underset{B}{\text{Max}}\,(U^{\text{f}}U^{\text{I}}) \tag{6}$$

This results in an outcome where the firm and inspector equally share the net benefits from underreporting the true level of emissions. The equilibrium bribe can be deduced to be

$$B = \frac{1}{2}(G(e) - G(e^{\text{h}}) - t(\hat{e}) + t(e^{\text{h}}) - \sigma(f(v,\theta) - I(v,\theta))) \tag{7}$$

Equation (7) reveals that the equilibrium bribe is declining in the fine imposed on the firm (that is, $f(v,\theta)$). Suppose that there are costs associated with auditing. Then, since the bribe is declining in $f(v,\theta)$), all corruption can be eliminated at an arbitrarily small audit cost, by levying a high fine which approaches infinity with an audit rate which is arbitrarily close to zero. To rule out this unrealistic case, we assume that the fines are bounded above by the after-tax income of each agent (that is, a limited liability constraint is imposed). Thus, the least-upper-bound of the firms' penalty is given by $(G(e) - t(\hat{e}))$, and that of the inspector by w.[14] For future reference, note that when fines are set at the maximum levels the equilibrium bribe is

$$B = \frac{1}{2}\,((G(e) - t(\hat{e}))(1 - \sigma) - G(e^{\text{h}}) + t(e^{\text{h}}) - \sigma w) \tag{8a}$$

The first-order condition (5) then simplifies to

$$\frac{\partial J}{\partial e} = \frac{\partial G}{\partial e}\,(1 - \sigma) = 0 \tag{8b}$$

The following useful properties of the equilibrium, illustrate certain important characteristics of the problem. The proofs are in Appendix A.

13 Having derived the equilibrium report, we can now provide a heuristic argument to demonstrate that an audit rate (λ) which is declining in $\hat{e}$ is optimal. More rigorous proofs of this result have been established elsewhere (see, for example, Border and Sobel, 1987). Suppose instead that $\partial\lambda/\partial\hat{e} \geq 0$. Compare the FOC in (4) when $\partial\lambda/\partial\hat{e} \geq 0$ with that when $\partial\lambda/\partial\hat{e} < 0$. Observe that the expected marginal cost of being prosecuted Γ, is lower when $\partial\lambda/\partial\hat{e} \geq 0$. Thus reported emissions and compliance is also lower in this case. Since pollution is easier to control with more honest reporting, it can be shown that an audit schedule with $\partial\lambda/\partial\hat{e} < 0$ is optimal for the regulator.

14 The assumption of an exogenously given upper bound on penalties, while widely employed in the literature, is clearly an unsatisfactory feature of models of corruption. The question as to why governments do not eradicate all misdemeanors by imposing draconian penalties is yet to be resolved. Fortunately the comparative statics below hold for non-maximal fines.

Lemma 1a

$$\frac{d\hat{e}}{d\tau} < 0 \; \forall \, f(v,\theta) \in (0,(G(e) - t(\hat{e}))] \; \textit{and} \; \forall \, I(v,\theta) \in (0, w]$$

Lemma 1a reveals that *ceteris paribus* an exogenous increase in the tax rate leads to lower reporting of emissions. Intuitively, a higher tax raises the costs of compliance and thus increases the payoffs from tax evasion. Accordingly, as the tax rate rises, reported emissions fall.

Lemma 1b

$$\frac{de}{d\tau} < 0 \; \forall \, f(v,\theta) \in (0,(G(e) - t(\hat{e}))) \; \textit{and} \; \forall \, I(v,\theta) \in (0, w]$$

Lemma 1b suggests that an increase in the tax rate leads to a reduction in actual pollution levels. This simply reflects the fact that higher taxes raise the costs of emitting pollution and hence result in lower discharges.

Lemma 1c

$$\frac{dv}{d\tau} = \frac{de}{d\tau} - \frac{d\hat{e}}{d\tau} > 0 \; \forall \, f(v,\theta) \in (0,(G(e) - t(\hat{e}))] \; \textit{and} \; \forall \, I(v,\theta) \in (0, w]$$

Lemma 1c reveals that higher taxes, by increasing the incentive to under-report, lead to lower levels of compliance.

Lemma 2a

$$\frac{d\hat{e}}{d\theta} \geq 0 \; \forall \, f(v,\theta) \in (0,(G(e) - t(\hat{e}))] \; \textit{and} \; \forall \, I(v,\theta) \in (0, w]$$

Lemma 2a summarizes the predictable result that an increase in the penalty rate, which raises the cost of a fine, dilutes the expected gains from corruption. Since the payoffs from corruption are lower, there is greater reporting of emissions.

Lemma 2b

$$\frac{d\hat{e}}{d\theta} > 0 \; \textit{if} \; \beta < \bar{\beta}, \; \textit{and} \; \frac{de}{d\theta} \leq 0 \; \textit{if} \; \beta \geq \bar{\beta},$$

$$\forall \, f(v,\theta) \in (0,(G(e) - t(\hat{e}))) \; \textit{and} \; I(v,\theta) \in (0, w]$$

where

$$\bar{\beta} = \frac{\lambda(\partial^2 F/\partial v \partial\theta)(\partial^2 t/\partial \hat{e}^2)}{\lambda \dfrac{\partial^2 F}{\partial v \partial \theta}\left(\dfrac{\partial \lambda}{\partial \hat{e}}\dfrac{\partial F}{\partial v} - F\dfrac{\partial^2 \lambda}{\partial \hat{e}^2} + \lambda \dfrac{\partial^2 F}{\partial v^2}\right) - \lambda \dfrac{\partial^2 F}{\partial \hat{e}^2}\dfrac{\partial \lambda}{\partial \hat{e}}\dfrac{\partial F}{\partial \theta} + \dfrac{\partial F}{\partial v}\dfrac{\partial \lambda}{\partial \hat{e}}\dfrac{\partial F}{\partial \theta}}$$

Finally, Lemma 2b reveals that if the prosecution rate (β) lies below a certain threshold level ($\bar{\beta}$), then a higher penalty rate (θ) results in an increase in emissions. This seemingly counterintuitive result arises for the following reason. First, recall that an increase in the penalty rate (θ)

induces an increase in reported emissions (Lemma 2a). Hence the firm is compelled to pay more tax on the reported emissions. The firm knows that when reported emissions ($\hat{e}$) increase, then *ceteris paribus*, the audit rate (λ) declines, so that the probability of being prosecuted falls. When β lies below the threshold level, the firm seeks to recover these costs by increasing actual emissions.

These results suggest that controlling emissions in this setting is likely to be a complicated process which depends on the responsiveness of e and $\hat{e}$ to each policy instrument. The regulator's problem is further complicated by the fact that policies must be based on reported rather than observed emission levels. The following section explores these issues in greater detail.

3. The welfare-maximizing policy response

This section analyzes the welfare-maximizing response of the regulator. A rational regulator who relies on reported emissions, will anticipate that the inspector and firm will collude to misreport emissions, whenever it is to their advantage. The regulator is therefore confronted with a problem of *hidden information*.[15] As is well known from the revelation principle, in these circumstances the regulator's optimal response can be restricted to the set of policies which induce the firm and inspector to honestly reveal the true level of emissions (Laffont and Tirole, 1998: 120). When truthful revelation is optimal for the firm and inspector, it is incentive compatible. Thus, we begin by deriving the incentive compatibility constraints.

Honest reporting will be individually rational for the firm if the payoffs from truthful revelation exceed those from corrupt behavior. Thus, for any given level of emissions, truthful revelation occurs if the costs to the firm associated with corruption exceed those from honest revelation. This implies that for a given $e \in [\underline{e}, \overline{e}]$

$$t(e) \leq t(\hat{e}) + B + \sigma f(v) \tag{9a}$$

Substituting for B, and rearranging, equation (9a) simplifies to

$$t(e) \leq t(\hat{e}) + \sigma F \tag{9b}$$

Let (9b) hold as an equality, then differentiating with respect to e (using the first-order condition in (4))

$$\frac{\partial F}{\partial v} = \frac{1}{\sigma}\frac{\partial t(e)}{\partial e} \tag{10a}$$

Equation (10a) asserts that truthtelling is incentive compatible when the marginal expected cost of a fine ($\sigma\ \partial F/\partial v$) equals the marginal increase in the tax burden ($\partial t(e)/\partial e$) . For future reference we note that when fines are at the maximum level this condition simplifies to

[15] Much of the literature on both environmental compliance and corruption appears to have ignored this aspect of the problem.

$$\frac{\partial G}{\partial e} = \frac{1}{\sigma}\frac{\partial t(e)}{\partial e} \tag{10b}$$

Thus, truthtelling occurs if the policies vary with e in the manner described by equations (10a) and (10b).[16]

Having defined the incentive compatibility conditions, we now specify the regulator's objective function. Pollution emissions cause external damage, described by the damage function, denoted $D(e)$. As usual, it is supposed that pollution damage is increasing in emission levels and convex: $\partial D/\partial e > 0$, $\partial^2 D/\partial e^2 > 0$. It is further assumed that there are costs associated with auditing, $a(\lambda)$, which increase with the probability of an audit at an increasing rate: $\partial a/\partial\lambda > 0$, $\partial^2 a/\partial\lambda^2 > 0$.

The regulator maximizes a utilitarian welfare function which is given by the total payoffs of all the agents in the model. Social welfare is thus given by the sum of profits, inspector's payoffs, government revenue from taxes and fines, less government spending on monitoring and inspectors wages, less the damage from pollution.[17] Upon simplification the welfare function is

$$W(e) = G - D(e) - a(\lambda) \tag{11a}$$

Since actual emissions are not observed, the regulator must maximize expected social welfare. As noted earlier, the regulator has some probability density function $\omega(e)$ for the unknown parameter (e), prior to receiving a report from the inspector. The cumulative distribution function is $\Omega(e)$. Then expected social welfare is

$$\int_{\underline{e}}^{\bar{e}} W(e)\omega(e)\mathrm{d}e \tag{11b}$$

By the revelation principle, the regulator maximizes expected welfare, subject to the constraint that truthtelling is incentive compatible.

We begin by outlining a general property of the equilibrium which allows discussion of the welfare-maximizing solution to be restricted to circumstances in which there is a feasible solution. Since auditing is costly, welfare is maximized by choosing the policy mix that minimizes the audit costs ($a(\lambda)$) of achieving any given level of emissions. Lemma 3 explores the implications of this observation.

[16] An identical condition holds for the inspector. Intuitively, the incentive compatibility constraint for the inspector is the same because in equilibrium the firm and inspector equally share the benefits of underreporting. Thus, without loss of generality, attention can be focused just on truthtelling by the firm.

[17] The firm's payoffs are $G - t - B - \sigma f(v,\theta)$, the inspector's utility is $w + B - \sigma I\ (v,\theta)$, the government's utility is $t + \sigma F(v,\theta) - a(\lambda) - w$ and finally pollution damage is $D(e)$. Summing these yields equation (11a). The usual utilitarian welfare function has the unappealing feature that payoffs from all sources (legal and illegal) are given equal weight. Thus, if total payoffs increase with corruption, bribery would be welfare improving. Moreover, as noted by a referee, such a function also implies that under usual assumptions about the utility function, theft by the poor from the rich can be welfare improving.

Lemma 3a

If the prosecution rate $\beta > \bar{\beta}$, *then the audit costs of achieving a given level of emissions are minimized if fines are set at the maximum feasible level* (*i.e.* $f(v,\theta) = (G(e) - t(\hat{e}))$ *and* $I(v,\theta) = w$)).

Proof Suppose that $\beta > \bar{\beta}$, but that $0 < f(v,\theta) < (G(e) - t(\hat{e}))$ and $0 < I(v,\theta) < w$. Then the fine can be increased and audit rate decreased such that the expected penalty for bribery (σF) is unchanged. Hence, by the FOC in equation (5) actual emissions can not increase. Since $\partial a/\partial\lambda > 0$, lowering the audit rate (λ) reduces audit costs (a). Since e does not increase, but audit costs fall as the fine rises, it follows that setting fines at their maximum levels (that is $f(v,\theta) = (G(e) - t(\hat{e}))$ and $I(v,\theta) = w$) minimizes the audit costs of achieving a given level of emissions, e.

Intuitively, if $\beta > \bar{\beta}$, the prosecution rate is sufficiently great so that high fines have the usual effect of deterring corrupt behavior. In this case it is clearly optimal for the regulator to set the fine at the highest feasible level in order to minimize on audit costs. Thus, there is no loss of generality in focusing on the case where fines are at the maximum level when the prosecution rate exceeds the threshold level (that is, $\beta > \bar{\beta}$).

Lemma 3b

If $\beta \leq \bar{\beta}$, *then the audit cost of achieving a given level of emissions are minimized if fines are set at* $f(v,\theta) = I(v,\theta) = 0$.

Proof Suppose that $\beta \leq \bar{\beta}$, but that $0 < f(v,\theta) < (G(e) - t(e))$ and $0 < I(v,\theta) < w$. Then the fine can be decreased and audit rate lowered such that actual emissions do not increase. Since $\partial a/\partial\lambda > 0$, lowering the audit rate reduces monitoring costs and raises welfare. It follows that lowering fines to the lowest feasible level $f(v,\theta) = I(v,\theta) = 0$ minimizes the audit costs of achieving a given level of emissions.

When $\beta \leq \bar{\beta}$, higher fines induce greater pollution and there is no effective mechanism for deterring corrupt behavior. In this situation it is optimal for the regulator to set the lowest feasible fine. More generally, it is shown in Proposition 4 below, that when $\beta \leq \beta$ there are no feasible policies for controlling emissions in this problem.

When $\beta > \bar{\beta}$ the regulator's problem is to maximize expected social welfare, subject to the constraints that truthtelling is incentive compatible and fines are at the upper bounds[18]

$$\max_{\tau} \int_{\underline{e}}^{\bar{e}} w(e)\omega(e)de \tag{12a}$$

subject to
$$\frac{\partial G}{\partial e} = \frac{1}{\sigma}\frac{\partial t(e)}{\partial e} \qquad \text{(Incentive compatibility)} \tag{12b}$$

[18] We do not include the usual individual rationality constraint which requires that the firm's payoffs exceed the next best alternative, so that there is an incentive to participate. This is because in the current context pollution damage may be so high that it is optimal to have zero emissions. The individual rationality constraint would rule out this outcome.

$$F = (G - t(e) + w) \qquad \text{(Maximum fines)} \qquad (12c)$$

As in Laffont and Tirole (1998: 64) equations (12a) and (12b) can be solved as an optimal control problem with an additional constraint (12c) on the problem. The solution is presented in Appendix B. The following propositions summarize the key properties of the welfare-maximizing policy response.

Proposition 1
If $\beta > \bar{\beta}$ *then*:
(a) by Lemma 3a in the welfare-maximizing equilibrium the fine is set at the maximum level.
(b) the emissions tax and audit rate are set such that the expected net marginal welfare gain from each of these policy instruments is equalized i.e.

$$\left((\sigma\frac{\partial G}{\partial e} - \frac{\partial D}{\partial e})\omega(e) + \frac{\partial \omega}{\partial e}\,W(e)\right)\frac{de}{d\tau} = \omega(e)\,(\frac{\partial a}{\partial \lambda}\,\frac{\partial \lambda}{\partial e}\,\frac{de}{d\tau})$$

Proof See Appendix B.

Part (a) of Proposition 1 is a direct consequence of Lemma 3a. It reflects the fact that the regulator can save on audit costs by setting fines at the highest feasible level. In part (b) of Proposition 1, the left-hand side defines the expected marginal welfare effects of higher taxes, while the right-hand side describes the expected marginal welfare effect of higher audit rates. Proposition 1b asserts that the optimum tax and audit regimes are set such that the expected net marginal benefit from a higher tax is equated to the expected increase in the marginal cost of auditing. To see why this is necessary, recall that higher taxes induce greater corruption, which in turn requires more auditing. Since auditing is costly, the optimum policy must trade-off the benefits from taxation against those from auditing. This contrasts with the conventional Pigouvian tax rule which requires that, absent corruption, the tax must be set equal to the net marginal damage from pollution. With the possibility of corruption, the regulator must take account of the additional inefficiencies associated with the incentive to evade higher taxes.

Proposition 2
If $\beta > \bar{\beta}$ *then the equilibrium tax burden rises at a diminishing rate with reported emissions (that is the tax schedule is strictly concave:* $\frac{\partial^2 t(e)}{\partial e^2} < 0$).

Proof See Appendix B.

Proposition 2 requires that in equilibrium the emission tax burden must rise at a decreasing rate with reported emissions. Intuitively, keeping the tax burden relatively low as pollution levels rise, provides the firm with a greater incentive to honestly report emissions when pollution levels are increasing. The *relatively* low tax paid by high polluters can be viewed as the additional rent that accrues to the polluter as a result of its informational advantage. Stated differently, asymmetric information forces the regulator to engage in costly auditing. To mitigate these costs the policy is distorted in favor of the polluter. The slowly rising tax schedule is thus a consequence of the truth telling constraint in the welfare maximization problem.

Proposition 3
If $\beta > \bar{\beta}$ *and if the tax burden rises sufficiently slowly with emissions, then the audit rate must decline at a decreasing rate with reported emissions (that is if the tax schedule is sufficiently concave, then the audit function must be convex* $\frac{\partial^2 \lambda}{\partial e^2} > 0$).

Proof See Appendix B.

When the audit rate declines at a decreasing rate (that is, is convex), low reports are audited relatively more intensively than high reports. This is necessary for two reasons. Firstly, note that with a concave tax schedule, the tax burden rises *relatively* steeply when pollution levels are low. There is therefore a relatively stronger marginal incentive to avoid the tax at low pollution levels. If underreporting is to be prevented at low pollution levels, the regulator must audit the low reports more intensively. This is clearly achieved with an audit schedule which is declining and convex in reported emissions. Secondly, recall that pollution damage is increasing at an increasing rate in emissions. Thus, when emission levels are high, the damage from pollution is high. Welfare considerations therefore dictate that emissions at high pollution levels continue to be monitored in order to strengthen the firm's incentive to comply with the regulation. This in turn requires that the audit rate does not decline too rapidly as reported emissions rise. Once again this can be achieved with a convex audit schedule.

Proposition 4
If $\beta < \bar{\beta}$ *then there is no feasible tax, penalty or audit rate which satisfies the necessary conditions for a maximum.*

Proof See Appendix B.

Intuitively, when β lies below a threshold level $\bar{\beta}$, the prosecution rate is low and there is no effective penalty available to the regulator to deter corrupt behavior. Since there is no workable mechanism to enforce compliance, there is little point in expending resources on auditing emissions. If auditing does not occur, there is complete evasion of the tax. Thus, the optimal solution is to abandon attempts to regulate emissions.

4. Conclusions

Global environmental problems in both developed and developing countries have been at the centre of controversy in a number of international forums such as the Rio Earth Summit, the greenhouse gas meetings and the 'Millennium Round' of trade talks in Seattle. However, corrective environmental policies are often rendered ineffective by corruption. Thus, an understanding of policies which promote corruption and ways to control it is of some importance. The existing literature on environmental compliance appears to have largely ignored the effects of corruption on environmental policy decisions. This paper has extended the literature by analyzing the problem of pollution control in a corrupt bureaucracy.

The model focused on the case where policy makers confront a stark choice between more stringent environmental regulations which increase opportunities for corrupt behavior, and greater enforcement which raises compliance costs. A number of new policy implications are suggested by the analysis. It was demonstrated that the optimal policy depends critically upon the efficiency of the judiciary, as defined by the prosecution rate. In highly inefficient judiciaries with a low prosecution rate, the commonly proposed expedient of harsher penalties for corruption was shown to cause greater pollution. The analysis reveals that in this situation there is little point in expending resources to control emissions unless the prosecution rate can be increased sufficiently.[19] In policy terms, increasing the prosecution rate would necessitate major institutional reform of the judiciary, which may be difficult to achieve since many of the factors which promote corruption are often those which preclude institutional reforms.

Where the judicial system is sufficiently effective in prosecuting offenders, the optimal response was shown to involve policies which combine efforts to reduce corruption together with those which lower emissions. Specifically, the welfare-maximizing policy requires that the net marginal benefit from the instruments for corruption control and pollution be equalized. This contrasts with the conventional (Pigouvian) approach which calls for the emission tax to be set equal to the marginal damage from pollution. In the current context, a higher tax creates stronger incentives to underreport, which in turn requires greater auditing. In the optimum solution the marginal benefits from taxation must therefore be traded off against those from auditing. This result suggests that the government may be severely restricted in its ability to control emissions, if auditing is sufficiently expensive. Moreover, it was shown that optimality necessitates a tax schedule which rises at a decreasing rate with reported emissions and an audit rate which declines at a decreasing rate with reported emissions. This combination lowers the incentive to pay bribes as emissions increase.[20] The results therefore suggest the need for multifaceted policies which tackle problems of corruption and pollution

[19] Anecdotal evidence of this phenomenon is not hard to find. For instance, the Wildlife Protection Society has documented cases of well-known poachers in India pending trial for over 40 major breaches of the Conservation Act, who continue to hunt and trade openly in wild animal parts (WPSI, 2000). Similarly, Pargal, Mani and Huq (1996) report that greater monitoring and reporting of pollution was associated with higher levels of emissions.

[20] It is of interest to note that this result is similar to that obtained in the literature on optimal taxation with costly monitoring and no corruption. For instance, Border and Sobel (1987) show that with monitoring costs the optimal tax increases and audit rates decline with income.

simultaneously. This contrasts with the approach taken by policy makers at both the domestic and international levels where problems of corruption and environmental damage are typically dealt with separately in an *ad hoc* and piecemeal manner (Desai, 1998).

It is worth noting that, while this paper has focused on the case of a pollution tax, the qualitative results may extend to any other instrument of environmental control, which raises compliance costs and requires monitoring by an agent. The increase in compliance costs creates an incentive to avoid the environmental control, while the need to monitor compliance creates an opportunity for agents to engage in corrupt behavior. Thus, the results might apply to the other main instruments of pollution control such as standards and pollution permits. It would be useful for future research to determine which of these instruments creates stronger incentives to engage in corrupt behavior. Another issue which warrants further research is the role of corruption and rent seeking in a renewable resource context. This is clearly important for the design of policies to control tropical deforestation and the preservation of endangered species.

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Appendix A

Totally differentiating the system in equations (4) and (5)

$$\begin{bmatrix} J_{\hat{e}\hat{e}} & J_{\hat{e}e} \\ J_{e\hat{e}} & J_{ee} \end{bmatrix} \begin{bmatrix} d\hat{e} \\ de \end{bmatrix} = - \begin{bmatrix} J_{\hat{e}\tau} \\ 0 \end{bmatrix} d\tau - \begin{bmatrix} J_{\hat{e}\theta} \\ J_{e\theta} \end{bmatrix} d\theta \qquad \text{(A1)}$$

where subscripts denote partial derivatives, thus

$J_{\hat{e}e} = \beta\left(-\frac{\partial\lambda}{\partial\hat{e}}\frac{\partial F}{\partial v} + \lambda\frac{\partial^2 F}{\partial v^2}\right) > 0$; if $f(v,\theta) \in (0, G(e) - t(\hat{e}))$, $I(v,\theta) \in (0,w)$.

With maximal fines (that is $f(v,\theta) = (G(e) - t(\hat{e}))$, $I(v,\theta) = w$),

$J_{\hat{e}e} = \beta\left(-\frac{\partial\lambda}{\partial\hat{e}}\frac{\partial G(e)}{\partial e}\right) > 0$. Further, $J_{\hat{e}\tau} = -\frac{\partial^2 t(\hat{e})}{(\partial\hat{e}\partial\tau)} < 0$ if $f(v,\theta) \in (0, G(e) - t(\hat{e}))$, $I(v,\theta) \in (0, w)$. With maximal fines $J_{\hat{e}\tau} = -\frac{\partial^2 t(\hat{e})}{(\partial\hat{e}\partial\tau)} + \beta\frac{\partial\lambda}{\partial\hat{e}}\frac{\partial t(\hat{e})}{\partial\hat{e}}\frac{\partial t(\hat{e})}{\partial\tau}$ < 0. (The sign of $J_{\hat{e}\tau}$ follows from the fact that it is assumed that:
$\frac{\partial^2 t(\hat{e})}{(\partial\hat{e}\partial\tau)} > 0$, $\frac{\partial\lambda}{\partial\hat{e}} < 0$, $\frac{\partial t(\hat{e})}{\partial\tau} > 0$.)

Moreover, $J_{\hat{e}\theta} = -\beta\frac{\partial\lambda}{\partial\hat{e}}\frac{\partial F}{\partial\theta} + \sigma\frac{\partial^2 F}{\partial v\partial\theta} > 0$ if $f(v,\theta) \in (0,G(e) - t(\hat{e}))$, $I(v,\theta) \in$ $(0, w)$. (The sign follows from the fact that $\frac{\partial^2 F}{\partial v\partial\theta} > 0$ $\frac{\partial\lambda}{\partial\hat{e}} < 0$, $\frac{\partial F}{\partial\theta} > 0$.)

Note that $J_{\hat{e}\theta} = 0$ if $f(v,\theta) = (G(e) - t(\hat{e}))$, $I(v,\theta) = w$, since in this case the fines are lump sum and set at the maximum amount.

Finally, $J_{e\theta} = -\sigma\frac{\partial^2 F}{\partial v\partial\theta} < 0$ if $f(v,\theta) \in (0,G(e) - t(\hat{e}))$, $I(v,\theta) \in (0, w)$.

To ensure that a unique maximum exists and is stable it is assumed that: $J_{\hat{e}e} < 0$, $J_{ee} < 0$ and that $|J_{e\hat{e}}| < |J_{\hat{e}\hat{e}}|$, $|J_{e\hat{e}}| < |J_{ee}|$.
Let $\Delta = J_{ee}J_{\hat{e}\hat{e}} - J^2_{\hat{e}e} > 0$ be the determinant of the 2 × 2 matrix in (A1).

Lemma 1a

$$\frac{d\hat{e}}{d\tau} = -\frac{J_{\hat{e}\tau}J_{ee}}{\Delta} < 0 \quad \forall f(v,\theta) \in (0, (G(e) - t(\hat{e}))],\ I(v,\theta) \in (0,w]$$

Lemma 1b

$$\frac{d\hat{e}}{d\tau} = \frac{J_{\hat{e}\tau}J_{e\hat{e}}}{\Delta} < 0$$

Lemma 1c

$$\frac{dv}{d\tau} = \frac{J_{\hat{e}\tau}(J_{e\hat{e}} + J_{ee})}{\Delta} > 0$$ (*the sign follows from the assumption that* $J_{ee} < 0$ *and that* $|J_{e\hat{e}}| < |J_{ee}|$)

Lemma 2a

$$\frac{d\hat{e}}{d\theta} = \frac{-J_{\hat{e}\theta}J_{ee} + J_{e\theta}J_{e\hat{e}}}{\Delta} \geq 0 \quad \forall f(v,\theta) \in (0, G(e) - t(\hat{e}))],\ I(v,\theta) \in (0, w]$$

Lemma 2b

$$\frac{de}{d\theta} = \frac{-J_{e\theta}J_{\hat{e}\hat{e}} + J_{e\theta}J_{e\hat{e}}}{\Delta} > 0 \text{ if } \beta < \bar{\beta}$$

$$\bar{\beta} = \frac{\lambda(\partial^2 F/\partial v\partial\theta)(\partial^2 t/\partial\hat{e}^2)}{\lambda\dfrac{\partial^2 F}{\partial v\partial\theta}\left(\dfrac{\partial\lambda}{\partial e}\dfrac{\partial F}{\partial v} - F\dfrac{\partial^2\lambda}{\partial\hat{e}^2} + \lambda\dfrac{\partial^2 F}{\partial v^2}\right) - \lambda\dfrac{\partial^2 F}{\partial\hat{e}^2}\dfrac{\partial\lambda}{\partial\hat{e}}\dfrac{\partial F}{\partial\theta} + \dfrac{\partial F}{\partial v}\dfrac{\partial\lambda}{\partial\hat{e}}\dfrac{\partial F}{\partial\theta}}$$

Appendix B

The Hamiltonian of the problem in (12a) and (12b) is

$$H = W(e)\omega(e) + \mu(e)\frac{1}{\sigma}\frac{\partial t}{\partial e} \tag{B1}$$

where $\mu(e)$ is the costate variable. Since there is an additional constraint (12c), the problem can be solved by forming the Lagrangean (see, Leonard and Long, 1992, Chapter 6)

$$L = H + \gamma(e)(F - (G - t(e) + w)) \tag{B2}$$

where $\gamma(e)$ is the Langrange multiplier.
The necessary conditions for a maximum are

$$\frac{dL}{d\tau} = \frac{dL}{de}\frac{de}{d\tau} + \frac{\partial L}{\partial\tau} = 0 \tag{B3}$$

$$\mu(e) = -\frac{\partial L}{\partial G} = -\omega(e) + \gamma(e) \quad \text{(B4)}$$

$$\frac{\partial G}{\partial e} = \frac{1}{\sigma}\frac{\partial t}{\partial e} \quad \text{(B5)}$$

$$\mathrm{G} - \mathrm{t}(e) + \mathrm{w} = \mathrm{F} \quad \text{(B6)}$$

Moreover we note that integrating both sides of (B4) yields

$$\mu(e) = -\Omega(e) + \int_{\underline{e}}^{\bar{e}} \gamma(e)\,\mathrm{d}e \quad \text{(B7)}$$

In a long-run steady state equilibrium $\dot{\mu} = 0$, thus in (B4) $\omega(e) = \gamma(e)$ and in (B7)

$$\mu(e) = -\Omega(e) + \int_{\underline{e}}^{\bar{e}} \omega(e)\,\mathrm{d}e = -\Omega(e) + \Omega(e) = 0$$

Proposition 1b

Expanding terms in (B3) yields

$$\left(\left(\sigma\frac{\partial G}{\partial e} - \frac{\partial D}{\partial e}\right)\omega(e) + \frac{\partial \omega}{\partial e}W\right)\frac{\mathrm{d}e}{\mathrm{d}\tau} - \omega(e)\left(\frac{\partial a}{\partial \lambda}\frac{\partial \lambda}{\partial e}\frac{\mathrm{d}e}{\mathrm{d}\tau} - \frac{\partial t}{\partial \tau}\right) = 0 \quad \text{(B8)}$$

Proposition 2

Rearrange (B5)

$$\lambda = \frac{1}{\beta}\left(\frac{\partial t/\partial e}{\partial G(e)/\partial e}\right) \quad \text{(B9)}$$

Differentiate with respect to e

$$\frac{\partial \lambda}{\partial e} = \frac{1}{\beta}\left(\frac{(\partial G(e)/\partial e)(\partial^2 t/\partial e^2) - (\partial G^2(e)/\partial e^2)(\partial t/\partial e)}{(\partial G(e)/\partial e)^2}\right) \quad \text{(B10)}$$

It has been assumed that $\frac{\partial \lambda}{\partial e} < 0$. *Moreover we have* $\frac{\partial t}{\partial e} > 0$, $\frac{\partial G}{\partial e} > 0$, $\frac{\partial^2 G}{\partial e^2} < 0$. *It follows from (B10) that a necessary condition for* $\frac{\partial \lambda}{\partial e} < 0$ *is* $\frac{\partial^2 t}{\partial e^2} < 0$.

Proposition 3

Upon further differentiation of (B10)

$$\frac{\partial^2 \lambda}{\partial e^2} = \begin{array}{l}(\partial G/\partial e)^2\,[((\partial G(e)/\partial e)(\partial^3 t/\partial e^3) - (\partial G^3(e)/\partial e^3)(\partial t/\partial e)) \\ -\,2(\partial^2 G/\partial e^2)((\partial G(e)/\partial e)(\partial^2 t/\partial e^2) - (\partial G^2(e)/\partial e^2)(\partial t/\partial e))]\end{array} \quad \text{(B11)}$$

Upon rearranging it can be seen that $\frac{\partial^2 \lambda}{\partial e^2} > 0$ *if* $\frac{\partial^2 t}{\partial e^2} < \Omega \equiv$

$$-\frac{(\partial t/\partial e)}{(\partial G/\partial e)} - \frac{(\partial G/\partial e)((\partial G/\partial e)(\partial^3 t/\partial e^3)) - (\partial G^3/\partial e^3)(\partial t/\partial e))}{2(\partial^2 G/\partial e^2)}.$$ [21]

[21] Note also that this condition always holds if it is assumed that the third derivatives of variables are sufficiently small or zero. This assumption is often evoked in the mechanism design literature as the third derivative appears to have no obvious economic interpretation in this context (Laffont and Tirole, 1998, Chapter 1).

Proposition 4

We now show that when $\beta < \bar{\beta}$*, then there is no feasible equilibrium tax. From Lemma 3b when* $\beta < \bar{\beta}$*, then it is optimal to set the fine rate* $\theta = 0$*. From equation (4) when* $\theta = 0$ *then* $\hat{e} = 0$*. Moreover from the incentive compatibility constraint (9b), if* $\theta = 0$*,* $\hat{e} = 0$ *and* $e > 0$ *then (9b) can never hold for any* $t(e) > 0$*. Thus, there is no positive tax which satisfies the incentive compatibility constraint in this case.*