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Water Distribution Network Reliability Estimation Using the First-Order Reliability Method

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Abstract

This paper utilizes the First-Order Reliability Method (FORM) for estimating water distribution network (WDN) reliability during critical loading conditions. The factors that are assumed to be uncertain include pipe roughness coefficients and nodal demands. FORM predictions of WDN nodal reliability compare closely with more accurate Monte Carlo Simulation (MCS) estimates. FORM is shown to automatically determine the critical node in the network in a reliable and efficient manner. The critical node reliability approximation to network reliability is evaluated and it is argued that the approximation is not appropriate in some cases. An improved FORM network reliability measure is proposed and shown to more closely approximate network reliability estimates when two failure modes are considered at the two worst nodes in the network. The findings of this paper should be considered in future WDN reliability-based optimization studies.

Introduction

Recent research in the design of water distribution networks (WDNs) has been focused on the estimation of WDN reliability. Failure events in WDNs can be classified as either mechanical or hydraulic in nature. Mechanical failures result from the failure of electromechanical components such as pump failure or pipe breakage. Hydraulic failures result from inadequate nodal flows and/or pressures being supplied due to factors such as increased demands or the degradation of pipe hydraulic capacities. Most WDN reliability studies focus on estimating either the hydraulic or mechanical reliability of WDNs. Some examples include the work by Goulter et al. (1986) and Su et al. (1987) who focused on the estimation of mechanical reliability and studies by Boa and Mays (1990) and Lansey et al. (1990) who estimated hydraulic reliability. Goulter (1995) provides a thorough review of reliability analysis methods and reliability-based optimization algorithms for WDNs. Some recent studies have attempted to estimate more representative measures of WDN reliability that consider both hydraulic and mechanical failures (e.g. Xu and Goulter 1999; Gargano and Pianese 2000).

The papers by Xu et al. (1998) and Xu and Goulter (1999) are particularly unique because they employ the First-Order Reliability Method (FORM) to estimate WDN reliability. Xu et al. (1998) establish that FORM predicts nodal hydraulic reliability reasonably well when compared with Monte Carlo Simulation (MCS) estimates of nodal hydraulic reliability. In the Xu and Goulter (1999) WDN reliability-based optimization study, nodal demands and the pipe hydraulic capacities were assumed to be independent random variables and the event-based reliability of trial networks was assessed under normal operating conditions and critical pipe failure scenarios.

This paper presents an improved approach for using FORM to estimate WDN reliability. Specifically, this paper will:

1. Present a simple and accurate approach for determining the critical node in the network when FORM is used for reliability estimation.
2. Demonstrate that the critical node reliability can be different than the network reliability.
3. Develop a new approximation of network reliability using FORM with two failure modes that is more accurate than the critical node approximation of network reliability.

Since this work is focused on the evaluation of FORM for WDN reliability analysis, a simple case study will be used where only event-based hydraulic reliability is considered. As in Xu and Goulter (1999), the reliability measure considered in this study is referred to as 'capacity reliability'. WDN capacity reliability refers to the probability that the minimum required nodal pressures are met, under the assumption that the required nodal demand flows are met, and is a function of the uncertain nodal demands and the uncertain degree to which pipe hydraulic capacities will be reduced over the design period.

Methods

Reliability Analysis. The performance of any engineered system can be expressed in terms of its load (demand) and resistance (capacity). If $X = (X_1, X_2, \ldots, X_n)^T$ is the vector of random variables that influences a system's load (L) and/or resistance (R), the performance function, $G(X)$, is commonly written as:

$$G(\boldsymbol{X}) = R - L \tag{1}$$

The failure (limit state) surface, $G = 0$, separates all combinations of $\boldsymbol{X}$ that lie in the failure domain (F) from those in the survival domain (S). Consequently, the probability of failure, p_f, is given as (Sitar et al. 1987):

$$p_f = \Pr\{X \in \mathrm{F}\} = \Pr\{G(X) < 0\} = \int\limits_{G(X)<0} f_X(x)\,\mathrm{d}x \tag{2}$$

where $f_X(\boldsymbol{x})$ is the joint probability density function (PDF) of $\boldsymbol{X}$. Reliability is then simply the complement probability to p_f.

In most realistic applications, the integral in Equation (2) is difficult to compute. Approximate solutions can be obtained by using a variety of techniques including Mean-value First-Order Second-Moment analysis (MFOSM) (Tung 1990), the Second-Order Reliability Method (SORM) (Madsen et al. 1986), MCS and FORM. The most accurate reliability estimation method is MCS with a very large number of realizations. Since all other reliability estimation techniques are generally developed to be more computational efficient than MCS, their accuracy should always be assessed in comparison with MCS benchmark solutions. MCS approximates the integral in Equation (2) by repeatedly generating random realizations of the variables in ***X*** and then evaluating the performance function in Equation (1) for each realization. The reliability obtained using MCS is then the ratio of the number of realizations for which $G(\boldsymbol{X}) > 0$ over the total number of MCS realizations evaluated. The MCS reliability estimate approaches the actual reliability as the total number of MCS realizations increases.

This research focuses on the use of FORM for approximating MCS predictions of WDN reliability. FORM was originally developed to assess the reliability of structures (Hasofer and Lind 1974), but has recently been used in water resources applications (Sitar et al. 1987; Xu and Goulter 1999). The objective of FORM is to compute the reliability index, β, which is then used to obtain the reliability, α, using:

$$\alpha = 1 - p_f = 1 - \Phi(-\beta) = \Phi(\beta) \tag{3}$$

where $\Phi(\,)$ is the standard normal CDF. In the n-dimensional space of the n random variables, β can be interpreted as the minimum distance between the point defined by the values of the n variable means (mean point) and the failure surface. Consequently, β may be thought of as a safety margin, as it indicates how far the system is from failure when it is in its mean state. The point on the failure surface closest to the mean point is generally referred to as the design point, or the most likely failure point. The reliability obtained using FORM is only an approximation, unless the performance function is linear and the random variables are all uncorrelated standard normal variables. The degree of non-linearity in the performance function, and hence the accuracy of FORM, is problem dependent. The reader is referenced to Madsen et al. (1986) or Xu and Goulter (1999) for a more complete description of FORM.

Reliability Measures. WDN reliability studies have generally concentrated on estimating the nodal, critical node and network reliability. Nodal and critical node capacity reliabilities are defined as the probability that the minimum required nodal pressures are met, under the assumption that the required nodal demand flows are met, for a specified node and for the node in the network that has the worst nodal capacity reliability, respectively. In contrast, network capacity reliability is defined as the probability that the minimum required pressures are met at *all* nodes. The type of reliability measured depends on the incorporation of these definitions in the performance function, any simplifying assumptions and can also depend on the reliability estimation technique used (e.g. FORM or MCS). In this work, MCS and FORM are used to estimate nodal, critical node and network capacity reliability.

Nodal Capacity Reliability. The performance function used to evaluate the capacity reliability of node i, by MCS or FORM is:

$$G_i(\boldsymbol{X}) = H_i(\boldsymbol{X}) - L_i^{\min} \tag{4}$$

where $H_i(\boldsymbol{X})$ is the head predicted at node i as a function of the vector of random nodal demands and pipe hydraulic capacities, ***X*** and $L_i^{\min}$ is the minimum specified head required at node i. In order to assess the reliability at all the nodes in the network using FORM, the performance function in Equation (4) must be specified for each node and a separate FORM computational procedure is then required. In contrast, MCS can be used to estimate the reliability at all nodes in the network in the same computational procedure since the performance functions at all nodes can be evaluated for each MCS realization. Therefore, the relative computational advantage of FORM over MCS diminishes quickly if the reliability at many or all nodes in the network is of interest.

Critical Node Capacity Reliability. The most basic way that FORM or MCS can be used to determine the critical node capacity reliability is to evaluate Equation (4) for each node in the network and then select the nodal reliability that is lowest. While this approach is no less efficient for MCS, Xu and Goulter (1999) recognized the increased computational burden imposed by FORM for estimation of multiple nodal capacity reliabilities. As a result they employed Equation (4) to find one measure of nodal capacity reliability at the most critical node in the network as determined by an additional stochastic hydraulic analysis during each evaluation of reliability. Although they observed that the increase in computational time is minimal and the approach seems reasonably accurate for this additional stochastic hydraulic analysis (Personal Communication, Chengchoa Xu), a more direct approach is proposed and tested here for identification of the critical node in the network. The following performance function can be defined for use with FORM in order to eliminate the need for any additional analysis to identify the critical node in the network:

$$G_c(\boldsymbol{X}) = \min(H_i(\boldsymbol{X}) - \mathrm{L}_i^{\mathrm{min}}),\ \ i = 1, 2, \ldots I \tag{5}$$

where $G_c(\boldsymbol{X})$ is the performance function at the node that is most critical with respect to meeting its corresponding minimum pressure requirement and I is the number of nodes with minimum pressure requirements. In this performance function, the location of the critical node is not fixed and instead FORM is left to converge to the critical node at the design point. This performance function is designed so that FORM finds the critical node and estimates the reliability at the critical node in one computational procedure.

Network Capacity Reliability. It is often convenient and sometimes more meaningful to estimate a single reliability measure that characterizes overall network performance. Previous approaches have proposed heuristic measures of network reliability such as the arithmetic mean or weighted average of all nodal reliabilities (e.g. Boa and Mays 1990). Another common heuristic measure of network reliability is the use of the critical node reliability as an approximation to the network reliability (e.g. Boa and Mays 1990, Xu and Goulter 1999). While these heuristics can provide reasonable estimates of network reliability, MCS can be used to directly estimate network reliability by treating WDNs as a series system in which failure at *any* node constitutes failure of the network to provide adequate heads.

In this study, a direct estimate of network capacity reliability can be found using MCS to evaluate the performance function in Equation (5). Even though both FORM and MCS can be used to evaluate the same performance function, the reliability measures estimated by each method are not the same. When Equation (5) is used as the FORM performance function, FORM converges to the critical node at the design point (i.e. the single most likely event to cause failure) and therefore estimates the capacity reliability for that node only. If the same performance function is defined for MCS reliability estimation, failure is defined with respect to meeting the minimum pressure requirement at the most critical node in the system for each MCS realization. The difference lies in the fact that the MCS technique measures the reliability with respect to all failure events simultaneously, instead of just measuring the reliability with respect to the most likely failure event.

At present, the only proposed approach for approximating network reliability using FORM is to assume it is approximately equal to the critical node reliability (Xu and Goulter 1999). In the context of WDN reliability-based design studies, making this approximation requires the assumption that the approximation will remain accurate for all possible network designs. The first problem with this is that the error in this approximation is non-conservative since the critical node reliability estimate only considers failure at a single node in the network. Events leading to failure at other nodes in the network that are independent of failure at the critical node are not considered by this heuristic. The second problem is that for design purposes it is generally impossible to determine the appropriateness of the approximation for all possible network designs. Therefore, this heuristic measure of network reliability should be avoided or used with great caution in reliability-based optimization studies. Consequently, if FORM is to be used to approximate network reliability more accurately in reliability-based optimization models, the network reliability estimate must be improved.

The FORM measure of network capacity reliability can be enhanced if an additional failure mode is considered. For example, two failure modes could represent the capacity reliability at two nodes of interest in the network. In any system with two failure modes, the total probability of failure, p_f, is:

$$p_f = p_{f1} + p_{f2} - p_{f12} = \Pr\{G_1 < 0\} + \Pr\{G_2 < 0\} - \Pr\{G_1 < 0 \text{ and } G_2 < 0\} \tag{6}$$

where p_{f1} and p_{f2} are the probabilities of failure due to failure modes 1 and 2, respectively; p_{f12} is the joint probability of failure for failure modes 1 and 2 and $G_1 = G(\boldsymbol{X}_1)$ and $G_2 = G(\boldsymbol{X}_2)$ are the performance functions for failure modes 1 and 2, respectively. The failure probabilities for the individual failure modes (p_{f1} and p_{f2}) can be obtained using Equation (2) while the joint probability of failure, p_{f12}, is given by Madsen et al. (1986) as:

$$\mathrm{p}_{f12} = \Phi(-\beta_1, -\beta_2; \rho_{12}) = \Phi(-\beta_1)\,\Phi(-\beta_2) + \int_0^{\rho_{12}} \psi(-\beta_1, -\beta_2; \mathrm{y})\,\mathrm{dy} \tag{7}$$

where $\Phi(\ ,\ ;\rho)$ is the CDF for a bivariate normal vector with zero mean values, unit variances and correlation coefficient ρ and $\psi(\ ,\ ;\rho)$ is the corresponding PDF. The integral in Equation (7) is generally obtained numerically. The correlation coefficient needed to evaluate this integral, ρ_{12}, is calculated using Madsen et al. (1986):

$$\rho_{12} = \frac{\boldsymbol{V}_1^{*T}\,\boldsymbol{V}_2^{*}}{\left|\boldsymbol{V}_1^{*}\right|\,\left|\boldsymbol{V}_2^{*}\right|} = \frac{1}{\beta_1\,\beta_2}\,\boldsymbol{V}_1^{*T}\,\boldsymbol{V}_2^{*} \tag{8}$$

where $\boldsymbol{V}_1^{*}$ and $\boldsymbol{V}_2^{*}$ are the design points in standard normal space for failure modes 1 and 2, respectively.

Although Equation (6) can be evaluated exactly when there are only two failure modes, only the bounds on the system probability of failure can be calculated when there are more than two modes of failure. In addition, the consideration of each additional mode of failure adds one more FORM computational procedure. Therefore, it is proposed that a more accurate point estimate of network capacity reliability for a reasonable increase in computational cost can be evaluated using FORM for reliability estimation in two failure modes (i.e. at two critical nodes in the network) in conjunction with Equations (6), (7) and (8) to estimate the series system probability of failure. The basic idea underlying this approach is to identify the second most critical failure node in addition to the most critical failure location. The two most critical nodes in the network are determined by first estimating the critical node capacity reliability using the performance function defined in Equation (5) and then estimating the capacity reliability for the next most critical node in the network according to the following performance function:

$$G_{2c}(X) = \min(H_i(X) - L_i^{\min}),\ \ i = 1, 2, \ldots I, \text{ and } i \neq k \tag{9}$$

where the performance function in the second mode of failure is Equation (9) and is defined as in Equation (5) except that node k, the critical node determined in the first failure mode, is not considered as a possible location of failure in the second mode of failure.

Based on Equation (6), the new system reliability measure, α_{net}, as estimated by FORM then becomes:

$$\alpha_{\text{net}} = 1 - \Pr\{\mathbf{X}_{1c} \in \mathrm{F} \cup \mathbf{X}_{2c} \in \mathrm{F}\} = 1 - p_{f1c} - p_{f2c} + p_{f1c2c} \tag{10}$$

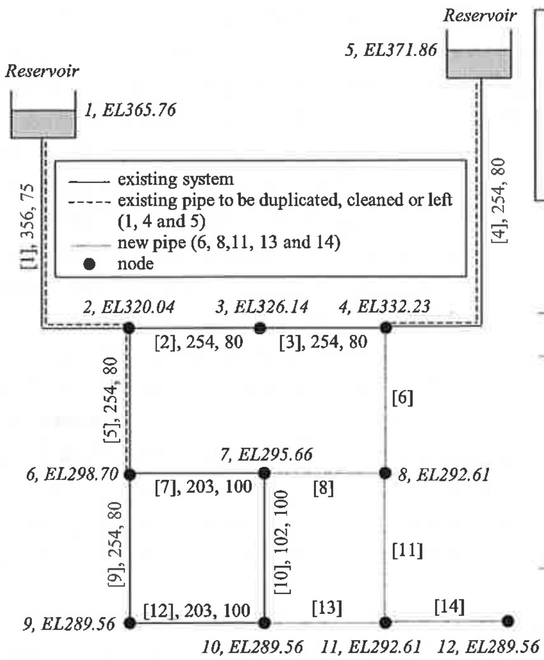
where the individual probabilities of failure at the first and second most critical nodes in the network are p_{f1c} and p_{f2c}, respectively, and are calculated from Equation (3) and the joint probability of failure between the two most critical nodes (p_{f1c2c}) is calculated from Equation (7).

Case Study

In this paper, a design problem that has been studied previously under deterministic conditions (Simpson et al. 1994) is utilized and transformed to be uncertain. The original design problem was to determine the required pipe sizes for an expansion to an existing WDN in order to minimize total pipe costs while still meeting the minimum nodal head requirements across the network under three different critical loading conditions. The three loading conditions considered represent the annual average peak-day, peak-hour demand levels, and two fire-loading demand cases that occur during the annual average peak-day demand levels. The physical layout of the design problem along with other network characteristics such as the head requirements and mean demand loading levels for each loading case, are summarized in Figure 1. Additional details regarding the design options can be found in Simpson et al. (1994).

Uncertainty Sources. The sources of uncertainty considered are the uncertain values of the Hazen-Williams coefficient, C, for each pipe and the uncertain nodal demand flows. The reliability in this study is calculated with respect to the uncertain conditions at the end of the design period under consideration and will thus, generally be the worst-case network reliability during any point in time during the design period. It is assumed that a random amount of degradation in the pipe hydraulic capacity (i.e. a random reduction in the Hazen-Williams coefficient, C, from present day values) will occur over the design period and that the nodal demand levels during the loading cases will exhibit the same random variability at the end of the design period as it would exhibit throughout the entire design period. The length of the design period is selected to be approximately 15 years for the purpose of this study. It is assumed that C values of each pipe will degrade by an average of 10% of their present day C values listed in

Figure 1 and the coefficient of variation of the reduction in *C* is 40%. For example, a pipe that has a present day *C* value of 120 is reduced on average by 12, with a standard deviation of 4.8, to a *C* value at the end of the design period of 108±4.8. The random amount of degradation in each pipe is assumed to be independent and is bounded for each pipe such that the minimum and maximum *C* values at the end of the design period are 60 and the present day *C* value, respectively. It should be noted that the reliability analysis program used in this work for the implementation of MCS and FORM automatically adjusts the PDF for each bounded random variable so that the total probability is equal to one. The nodal demand levels in all three critical loading cases are assumed to be normally distributed, with means as listed in Figure 1 and a COV of 40% for all non-fire demand nodes. The fire demand nodes considered are node 7, in loading case 2, and node 12, in loading case 3, and both are assumed to have a reduced COV of 5%. All nodal demands are bounded to be greater than 0 and are assumed to be uncorrelated.



[1], 356, 75 [pipe number], diameter (mm), present day Hazen-Williams roughness coefficient (*C*)

2, EL320.04 node number, node elevation (m)

Note: 1. All pipe lengths are 1609m, except pipe [1] and pipe [4] which are 4828m and 6437m, respectively.
2. *C* is 120 for new and cleaned pipes.

	Demand Pattern 1		**Demand Pattern 2**		**Demand Pattern 3**	
Node	Mean *Q* (L/s)	H_{min} (m)	Mean *Q* (L/s)	H_{min} (m)	Mean *Q* (L/s)	H_{min} (m)
2	12.62	28.18	12.62	14.09	12.62	14.09
3	12.62	17.61	12.62	14.09	12.62	14.09
4	0	17.61	0	14.09	0	14.09
6	18.93	35.22	18.93	14.09	18.93	14.09
7	18.93	35.22	82.03	10.57	18.93	14.09
8	18.93	35.22	18.93	14.09	18.93	14.09
9	12.62	35.22	12.62	14.09	12.62	14.09
10	18.93	35.22	18.93	14.09	18.93	14.09
11	18.93	35.22	18.93	14.09	18.93	14.09
12	12.62	35.22	12.62	14.09	50.48	10.57

Figure 1. Layout of the two-reservoir network upgrade problem under uncertainty

Reliability Model. The reliability model needed to evaluate FORM and MCS nodal and network capacity reliabilities was developed in FORTRAN by linking a hydraulic network solver with a reliability analysis program. For each reliability estimate, the Wadiso hydraulic network simulation program (Gessler and Waliski 1985) was used to simulate the hydraulic system for each vector of random variable values generated by FORM or MCS. Wadiso assumes that the nodal demands are met and solves for the resultant nodal heads across the network. The original version of the program was modified so that it can be repeatedly executed without the use of an input file. The accuracy of this modified version of Wadiso was verified against a standard hydraulic simulation package (EPANET2). A versatile and well-tested reliability analysis program called RELAN (Foschi et al. 1993) was used to implement the FORM and MCS procedures.

This paper is focused on evaluating the proposed reliability measures and therefore does not involve any optimization. Instead, six different potential designs were selected for all the analyses presented such that the resultant network capacity reliabilities span a reasonable range of reliabilities. Three critical loading cases per design lead to nodal and network reliability estimates for 18 network design conditions.

Results

Initial testing was done to determine the number of MCS realizations required to generate accurate MCS benchmark reliabilities. MCS nodal and network capacity reliability estimates using 100000 realizations converged to within approximately 0.001 of the MCS reliabilities found using 500000 realizations. Thus, all MCS reliabilities presented were generated using 100000 realizations. A comparison of the nodal reliabilities (Equation (4)) obtained using MCS and FORM for the 18 design cases showed that, at worst, the difference in the FORM and MCS estimates of nodal reliability was 0.017. When the nodal reliabilities were significantly different than 1.0, differences typically ranged between 0.005 and 0.01. Therefore, the case study was deemed appropriate for evaluating the new FORM reliability measures.

The FORM nodal capacity reliability predictions discussed above were evaluated to identify the critical node location and its corresponding reliability estimate. These critical node locations and their corresponding reliabilities were then compared to the FORM critical node locations and reliabilities found using Equation (5). In addition, this comparison also includes an evaluation of the increase in computational cost of using Equation (5) in terms of the number of performance function evaluations required by FORM. In 17 of the 18 cases, the use of Equation (5) led to the correct identification and reliability estimate for the critical node. In the one case where the correct critical node was not identified, the third most critical node was identified and the difference in reliabilities between the critical and third most critical node was less than 0.002. The average and worst observed increase in computational cost associated with using Equation (5) was observed to be 11.4% and 31.0%, respectively. This small average increase in computational cost was deemed a reasonable tradeoff for the increased simplicity and consistency in which the proposed performance function definition allows FORM to accurately determine the critical node.

In order to evaluate the accuracy of the critical node capacity reliability approximation of network capacity reliability, the MCS predictions of nodal capacity reliability were evaluated to determine the MCS critical node capacity reliabilities for all cases. Equation (5) was used to generate the actual MCS estimates of network capacity reliability, which are then compared to MCS critical node capacity reliabilities. Results show that for eleven of 18 cases the approximation is quite reasonable since the approximations fall within 0.001 of the actual network capacity reliability. Even though the results for the remaining seven cases remain close approximations, the differences in reliabilities of greater than 0.001 show that these two quantities are in fact different. The largest discrepancy in the approximation occurred for a case where the actual network capacity reliability of 0.943 is overestimated by 0.015 with a critical node capacity reliability of 0.958. Although this error may initially seem small, another way to consider the significance of such an error is to consider the increase in cost that would be incurred in a WDN design problem if the design network reliability level was to be increased from 0.958 instead of 0.943. If this cost increase is significant then the approximation error created by using the critical node reliability is also significant.

Given that the critical node approximation to network capacity reliability was not accurate for some of the cases in this study, the next step was to determine if using Equation (10) in conjunction with FORM estimates of the reliability at the two most critical nodes (using two modes of failure) would improve the FORM estimate of network capacity reliability. Table 1 presents the new FORM approximations to network capacity reliability in comparison with MCS estimates of network capacity reliability and FORM critical node reliability estimates. Many cases show little or no absolute improvement of the FORM estimate of network capacity reliability as seen by comparing columns (2) and (3) in Table 1. However, if the relative reduction in the FORM approximation errors is considered (last column in Table 1) then it becomes clearer that the new FORM approximation of network capacity reliability proposed here resulted in a significantly more accurate estimate of network reliability by FORM. In fact, the average reduction in error for the eight cases where the reduction is greater than 0% is 35.2%.

Table 1. Comparison of alternative measures of network capacity reliability.

Design (Loading Case)	MCS Network Capacity Reliability (1)	FORM Network Capacity Reliability (2)	FORM Critical Node Capacity Reliability (3)	Percent Reduction in FORM Approximation Error[A]
A (1)	1.000	1.000	1.000	-
A (2)	0.981	0.982	0.982	0.0%
A (3)	1.000	1.000	1.000	-
B (1)	0.995	0.996	0.998	66.7%
B (2)	0.943	0.946	0.958	80.0%
B (3)	0.995	0.996	0.996	0.0%
C (1)	0.999	0.999	0.999	-
C (2)	0.924	0.924	0.924	-
C (3)	0.996	0.996	0.996	-
D (1)	0.954	0.957	0.957	0.0%
D (2)	0.804	0.822	0.826	18.2%
D (3)	0.818	0.835	0.835	0.0%
E (1)	0.982	0.985	0.987	40.0%
E (2)	0.689	0.709	0.719	33.3%
E (3)	0.948	0.954	0.957	33.3%
F (1)	0.911	0.919	0.919	0.0%
F (2)	0.605	0.621	0.622	5.9%
F (3)	0.524	0.549	0.55	3.8%

A. Reduction in error = 100%[1.0 –{(2) – (1)}/{(3)-(1)}]

Conclusions and Future Work

This paper has demonstrated a number of significant improvements in the approach to WDN reliability estimation that should be considered in future WDN reliability-based optimization studies. A new, accurate and efficient method for identifying the most critical node in the network has been developed when FORM is used for reliability estimation. It is argued that the critical node capacity reliability approximation to network capacity reliability estimation may be an unreasonable general assumption in WDN reliability-based optimization studies. In order to improve this approximation in studies utilizing FORM for reliability estimation, a new, relatively efficient and more accurate FORM approximation to network capacity reliability is presented that considers two modes of failure at the two most critical nodes in the network.

Specific future work regarding the approaches proposed in this case study will be its incorporation into reliability-based optimization models. Additional future work in this area relates to expanding the WDN reliability estimation approach used here. Specifically, other FORM techniques for accurate estimation of the network capacity reliability could be developed since the method proposed here is still an approximation. Additional WDN variables could be considered uncertain, such as reservoir or tank levels, and correlations between the uncertain variables should be evaluated. The new FORM reliability measures demonstrated here need to be tested on larger WDN design problems. Work is also required to further assess the overall suitability of FORM for WDN reliability estimation.

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