E. C. Bullard

29th December, 1958.

Dear Teddy,

I have been sorting out the smoothing formulae a little more thoroughly, and think you might like to have a copy made of the enclosed sheet so as to have available in the Department what is rather tedious to reproduce anew.

For each formula is given the series of multipliers of adjacent terms in the series to be smoothed, so that for the 5-point formula one multiplies five successive values by -3, 12, 17, 12, -3, dividing by 35. Then for this series the opacity for random variance is one less than the ratio of the central coefficient to the divisor. As about one-fifth of the random variants will be in wave lengths exceeding 10, one would not wish to press this opacity above 80%, if, as in my case, the second harmonic giving the wave form is about 10 or a trifle over. On the right I give the coefficients for expressions in δ^4, δ^6, etc., and for this series the coefficients of δ^4 doubled and multiplied by the angle α^4 give the opacity for small values of α.

In the second series it is the coefficients of δ^6 which, when doubled, give the coefficient of α^6 in the corresponding expression for the opacity.

I mentioned that this second series gave a sharper cut-off, though it has the grave disadvantage of involving more terms, and the little table at the foot of the page gives the opacity for the 15-point formula,(which was the only one I was much tempted to think might be better than the 11-point formula that I actually used) over the range of the cut-off between 6 and 12 year periods. This is just an exhibit to show how sharp it is.

If one seriously wanted to get it sharper, the process I am discussing has the grave drawback that the number of terms required is getting to be largish compared with the length of the series, and as this is always finite, any more exacting process would have to take its value into account. I believe this would best be done by using the Fourier sub-multiples, e.g. with a series of 119 terms (an odd number being a trifle more convenient than an even one) one could tabulate the 59 sines and cosines of $n\pi/59$ round the circle from any origin, and calculate the two components of (let us say) the first 14 harmonics, which ~~our~~ on re-plotting would give an absolute cut-off between periods of 119/14 and 119/15, retaining the components corresponding with longer periods intact. There would, I suspect, also be a rather troublesome value for rate of change linear with time corresponding with the periodic time 119/0; but I am not quite clear about that.

However, I suppose even an electronic computor would not welcome storing 118 trigonometric functions in its memory, and without that aid the job would be tedious, and for personal computation, I imagine, exceedingly confusing.

Of course, I suppose all these procedures would give in fact very similar pictures!

Sincerely yours,

Enc.

Sir Edward C. Bullard, F.R.S.

OPACITY

5 point	-3	12	17	. . . . /35			18/35	51.4285%	$1-\frac{3}{35}\delta^4$
7 point	-2	3	6	7 . . . . . . /21			2/3	66.6667%	$1-\frac{3}{7}\delta^4-\frac{2}{21}\delta^6$
9 point	-21	14	39	54	59 . . . /231		172/231	74.4589	$1-\frac{9}{7}\delta^4-\frac{2}{3}\delta^6-\frac{1}{11}\delta^8$
11 point	-36	9	44	69	84	89 . /429	340/429	79.2541	$1-3\delta^4-\frac{8}{3}\delta^6-\frac{9}{11}\delta^8-\frac{12\delta^{10}}{\cancel{145}\;143}$

7 point	5	- 30	75	131						/ 231
9 point	15	- 55	30	135	179					/ 429
11 point	18	- 45	- 10	60	120	143				/ 429
13 point	110	- 198	- 135	+110	+ 390	+ 600	+ 677.			/ 2431
15 point	2145	-2860	-2937	-165	3755	7500	10125	11063	. . .	/46189
17 point	195	- 195	- 260	-117	135	415	660	825	833 .	/ 4199

		opacity	
7 point	100/231	43.2900 %	$1+\frac{5\delta^6}{231}$
9 point	250/429	58.2751 %	$1+\frac{5\delta^6}{33}+\frac{5\delta^8}{143}$
11 point	2/3	66.6667 %	$1+\frac{20\delta^6}{33}+\frac{45\delta^8}{143}+\frac{6\delta^{10}}{143}$
13 point	$\frac{1754}{2431}$	72.1514 %	$1+\frac{20\delta^6}{11}+\frac{225\delta^8}{\cancel{175}\;143}+\frac{6\delta^{10}}{13}+\frac{10\delta^{12}}{221}$
15 point	$\frac{2702}{3553}$	76.0484	$1+\frac{50}{11}+\frac{75}{13}+\frac{36}{13}+\frac{10}{17}+\frac{15}{323}$
17 point		76.9712	$1+10+\frac{225}{13}+12+\frac{70}{17}+\frac{225}{323}+\frac{15}{323}$