23rd December, 1958.

Dear Teddy,

Many thanks for your note and for the graph, which really shows very well what the smoothing formulae are doing. The more elaborate formula has a cut-off at about an octave lower frequency than the simple one you use, which is the first of a series using 5, 7, 9, 11, etc., points of which mine is the fourth. They all have opacities varying as α^4 for small α, where α is the angular change for one year, and therefore are sufficiently transparent for long waves.

You can get a sharper cut-off with limiting opacity varying as α^6 from a series starting with the simple ~~subtraction~~ addition of one-fortieth of the sixth difference, but one might have to go to rather a lot of terms in this series in order to wipe out the massive disturbance due to chance which occurs in the short waves.

I hope I shall be seeing you over the holiday.

Sincerely yours,

<u>Sir Edward Bullard, F.R.S.</u>

Smoothing by subtracting $\frac{1}{12}\delta^4$

$$\delta^4 = x_{n+2} - 4x_{n+1} + 6x_n - 4x_{n-1} + x_{n-2}$$

$$\bar{x}_n = \text{smoothed } x_n = \frac{-x_{n+2} + 4x_{n+1} + 6x_n + 4x_{n-1} - x_{n-2}}{12}$$

If this is applied to

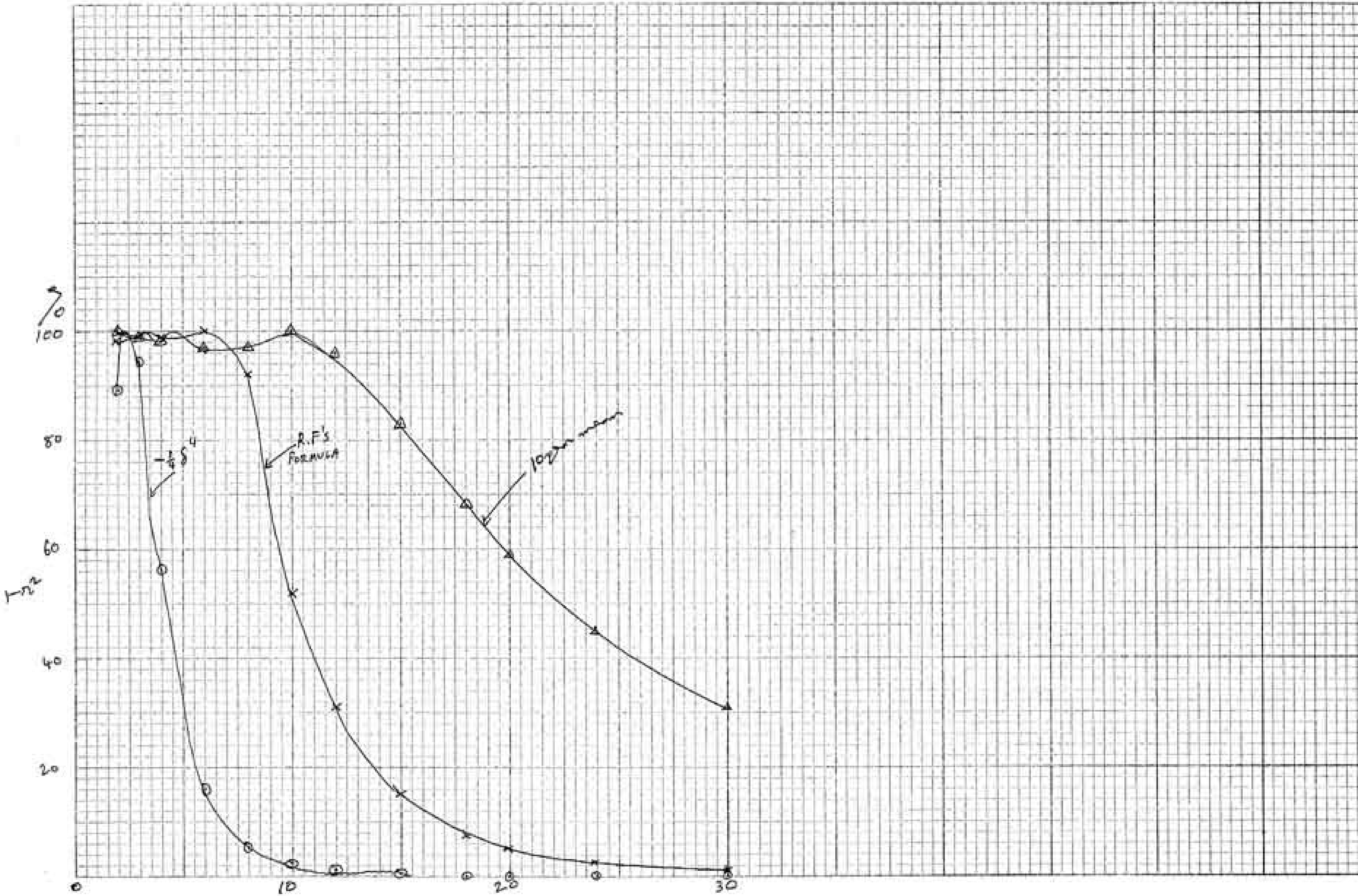
$$x_n = \cos\alpha n$$

the smoothed x_n has amplitude

$$r = \frac{6 + 8\cos\alpha - 2\cos 2\alpha}{12}$$

Taking $\alpha = \frac{360^\circ}{T}$ this gives

T	r	$100(1-r^2)$ %
30	0.99984	.032
24	.99961	.077
20	.99920	.16
18	.99879	.24
15	.99758	.48
12	.99402	1.2
10	.98784	2.4
8	.97140	5.6
6	.91667	16
4	.66667	56
3	.25000	94
2	−.33333	89



%
100
80
60
40
20
$1-\eta^2$
$-\frac{1}{4}\delta^4$
R.F's FORMULA
0
10
20
30