A graph decomposition-based approach for water distribution network optimization

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Received 31 May 2012; revised 27 February 2013; accepted 2 March 2013; published 24 April 2013.

[1] A novel optimization approach for water distribution network design is proposed in this paper. Using graph theory algorithms, a full water network is first decomposed into different subnetworks based on the connectivity of the network's components. The original whole network is simplified to a directed augmented tree, in which the subnetworks are substituted by augmented nodes and directed links are created to connect them. Differential evolution (DE) is then employed to optimize each subnetwork based on the sequence specified by the assigned directed links in the augmented tree. Rather than optimizing the original network as a whole, the subnetworks are sequentially optimized by the DE algorithm. A solution choice table is established for each subnetwork (except for the subnetwork that includes a supply node) and the optimal solution of the original whole network is finally obtained by use of the solution choice tables. Furthermore, a preconditioning algorithm is applied to the subnetworks to produce an approximately optimal solution for the original whole network. This solution specifies promising regions for the final optimization algorithm to further optimize the subnetworks. Five water network case studies are used to demonstrate the effectiveness of the proposed optimization method. A standard DE algorithm (SDE) and a genetic algorithm (GA) are applied to each case study without network decomposition to enable a comparison with the proposed method. The results show that the proposed method consistently outperforms the SDE and GA (both with tuned parameters) in terms of both the solution quality and efficiency.

Citation: Zheng, F., A. R. Simpson, A. C. Zecchin, and J. W. Deuerlein (2013), A graph decomposition-based approach for water distribution network optimization, *Water Resour. Res.*, *49*, 2093–2109, doi:10.1002/wrcr.20175.

1. Introduction

[2] The optimization of water distribution network (WDN) design has been investigated over the past few decades, and a number of optimization techniques have been developed to tackle WDN optimization problem. These include linear programming (LP) [*Alperovits and Shamir*, 1977], nonlinear programming (NLP) [*Fujiwara and Khang*, 1990], and evolutionary algorithms (EAs) [*Dandy et al.*, 1996; Montesinos *et al*., 1999; *Reca and Martínez*, 2006; *Maier et al*., 2003; *Tolson et al*., 2009; *Suribabu*, 2010; *Zheng et al*., 2013a]. However, it has been found that each optimization algorithm has its own advantages and disadvantages.

[3] For LP and NLP, optimal solutions can be located efficiently, while only local minimums are provided. EAs are able to find good quality solutions but are computationally expensive. A number of advanced methods have been proposed to reduce the computational intensity required by EAs in terms of WDN optimization [*van Zyl et al*., 2004; *Tu et al*., 2005; *Keedwell and Khu*, 2005; *Reis et al*., 2006]. Combining optimization techniques with water network decomposition is one of those advanced methods.

[4] Normally, a WDN can be viewed as a connected graph $G(V,E)$, where V is a set of links and E is a set of nodes in the WDN. Thus, it is natural to introduce graph theory algorithms to facilitate WDN analysis. Traditionally, graph theory was used for water network connectivity and reliability analysis. *Gupta and Prasad* [2000] used linear graph theory for the analysis of the pipe networks. *Deuerlein* [2008] proposed a graph theory algorithm to decompose a WDN into forests, bridges, and blocks. This method provides a tool to simplify complex WDNs and provides a better understanding of the interactions between their different parts of the network.

[5] In terms of WDN optimization, *Kessler et al*. [1990] developed a graph theory based algorithm to optimize the design of WDNs. In their work, the design process consisted of three distinct stages. In the first stage, alternative paths were allocated using graph theory algorithms. In the second stage, the minimum hydraulic capacity (diameters) of each path was determined using an LP model. In the third stage, the obtained solution from the second stage was tested by a network solver for various demand patterns.

[6] *Sonak and Bhave* [1993] introduced a combined graph decomposition-LP algorithm for WDN design. In

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0043-1397/13/10.1002/wrcr.20175

this combined algorithm, all the trees of the looped WDN were first identified by a graph theory algorithm and optimized by a LP, allowing the global optimum tree solution to be located. The final optimal solution for the original WDN was then determined by assigning the chords of the global optimum tree the minimum allowable pipe diameters. *Savic and Walters* [1995] used graph theory to partition a water network into "tree" and "cotree" to enable an optimization problem that involved minimizing the heads by setting regulation valves.

[7] *Kadu et al*. [2008] proposed a genetic algorithm (GA) combined with a graph theory algorithm to optimize water distribution systems. In their method, graph theory is used to identify the critical path for each node in order to reduce the search space for the GA. *Krapivka and Ostfeld* [2009] proposed a coupled GA-LP scheme for the least cost pipe sizing of water networks. A spanning tree identification algorithm was introduced in their work. *Zheng et al*. [2011] proposed a combined NLP-DE algorithm to optimize WDNs. In this algorithm, a graph theory algorithm was first used to identify the shortest-distance tree for the original whole WDN. Then, an NLP was implemented to optimize the tree network. The optimal solution obtained from the NLP optimization was finally utilized to seed a DE to optimize the original whole network.

[8] Improvements in terms of efficiency and solution quality have consistently been reported by researchers when these optimization techniques are combined with graph theory algorithms and applied to WDN case studies. It was observed that, for the existing graph theory based optimization techniques, graph theory is normally used to identify the critical path or the spanning tree for the WDN in order to facilitate optimization.

[9] For the proposed method here, a complete WDN is decomposed into subnetworks (rather than spanning trees) based on the connectivity of the network's components. The resulting subnetwork may consist of a single block, bridges to this block, and/or trees connected to this block. For relatively simple networks (such as networks that have only one block and multiple trees attached to this block (case studies 2 and 3 in this paper)), the trees can be viewed as subnetworks. The subnetwork containing the water supply node (reservoir) is designated the root subnetwork. The definitions of block, bridge, and tree for the water network are given by *Deuerlein* [2008], who described a block in a WDN as a maximal biconnected subgraph; a bridge is a link joining two disconnected parts of a graph; and a tree is a connected subgraph without any circuits or loops.

[10] After the subnetworks have been identified, each one is represented as an augmented node, and these augmented nodes are connected using *directed links* to form a directed augmented tree (AT), in which the directed links are used to specify the subnetwork optimization sequence. In order to improve the efficiency of the optimization process, a preconditioning approach is developed to approximately optimize the subnetworks in order to produce an approximate optimal solution for the original full network. The obtained approximate solution is able to specify promising regions within the entire search space. A final optimization method is then used to exploit these promising regions in order to generate further improved solutions for the original full network.

[11] The proposed optimization method presented in this paper is suited for the water networks where the graph decomposition can be applied and the WDN has no pumps and multiple reservoirs. The outcome of the proposed method is a significant improvement over the state of the art for designing common WDNs, and at the same time, is a starting point for future improvements to be applied to any WDN configuration. The details of the proposed methodology are given later.

2. Formulation of WDN Optimization Problem

[12] Typically, a single-objective optimization of a WDN is the minimization of system costs (pipes, tanks, and other components) while satisfying head constraints at each node. In this paper, the proposed graph decomposition based optimization method is verified using WDN case studies with pipes only. Thus, the formulation of the WDS optimization problem can be given by

$$\text{Minimize} \qquad F = a\sum_{i=1}^{np} D_i^b L_i, \tag{1}$$

[13] Subject to:

$$H_k^{\min} \leq H_k \leq H_k^{\max} \qquad k = 1, 2, ..., nj, \tag{2}$$

$$G(H_k, D) = 0, \tag{3}$$

$$D_i \in \{A\}, \tag{4}$$

where F is the network cost that is to be minimized [*Simpson et al*., 1994]; D_i is the diameter of the pipe i; L_i is the length of the pipe i; a, b are specified coefficients for the cost function; np is the total number of pipes in the network; nj is the total number of nodes in the network; $G(H_k, D)$ is the nodal mass balance and loop (path) energy balance equations for the whole network, which is solved by a hydraulic simulation package (EPANET2.0 in this study); H_k is the head at the node $k = 1,2 \ldots, nj$; $H_k^{\min}$ and $H_k^{\max}$ are the lower and upper head limits at the nodes; and A is a set of commercially available pipe diameters.

3. Methodology

[14] Four steps are involved in the proposed method for optimizing a WDN.

[15] Step 1: The subnetworks for the full WDN that is being optimized are identified using a graph decomposition algorithm.

[16] Step 2: A directed AT is built for the original full WDN. In the AT, the subnetworks appear as augmented nodes connected by directed links. The direction of the directed links in the AT determines the subnetwork optimization sequence in the proposed method.

[17] Step 3: The subnetworks are then preconditioned using a DE algorithm to produce an approximate optimal solution for the original full network.

[18] Step 4: The subnetworks are further optimized by a DE algorithm based on the approximate optimal solution obtained in Step 3.

[19] The details of each step are as follows.

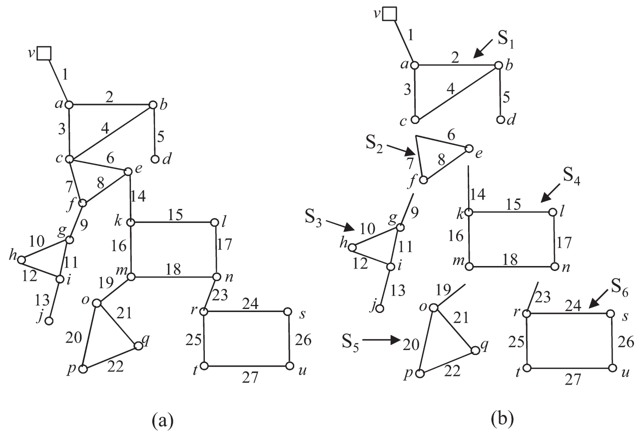

Figure 1. An example of 27-pipe water network decomposition. (a) The original water network (G). (b) The proposed decomposition results (S).

3.1. Subnetwork Identification for the Full Water Network (Step 1)

[20] *Deuerlein* [2008] proposed a graph theory algorithm to decompose a water network graph (G) into forest, blocks, and bridges according to its connectivity properties. In the method proposed here, however, the original network graph (G) is decomposed into a series of subnetworks (S). Each of the subnetworks may consist of one block, bridges to this block and trees attached to this block if applicable, or purely trees (if blocks are not applicable)

[21] Figure 1 illustrates the decomposition results of a water network using the proposed new method. For the WDN (G) given in Figure 1a, six subnetworks are identified specified as follows by a set of nodes and pipes, including $S_1 = \{a, b, c, d, v, 1, 2, 3, 4, 5\}$, $S_2 = \{e, f, 6, 7, 8,\}$, $S_3 = \{g, h, i, j, 9, 10, 11, 12, 13\}$, $S4 = \{k, l, m, n, 14, 15, 16, 17, 18\}$, $S_5 = \{o, p, q, 19, 20, 21, 22\}$, and $S_6 = \{r, s, t, u, 23, 24, 25, 26, 27\}$. S_1 is denoted as a root subnetwork as it includes the supply source node v of the original water network.

[22] As shown in Figure 1b, each subnetwork contains one and only one block, bridges to this block if applicable, and the trees attached to this block if applicable. The subnetworks overlap at some nodes as can be seen from Figure 1, i.e., $S_1 \cap S2 = c$, $S_2 \cap S3 = f$, $S_2 \cap S4 = e$, $S_4 \cap S5 = m$, and $S_4 \cap S6 = n$. In this study, nodes c, f, e, m, and n are denoted as subnetwork cut nodes (C), i.e., $C = \{c, f, e, m, n\}$. A depth first search (DFS) is employed to identify subnetwork cut nodes [*Tarjan*, 1972; *Deuerlein*, 2008] to enable network decomposition.

3.2. Directed AT Construction for the Original WDN (Step 2)

[23] In order to assist in visualizing the proposed optimization method, the decomposed water network G is reconstructed as a directed AT by imagining each of the subnetworks as an *augmented node* and connecting the augmented nodes using directed links. The directed augmented tree AT of water network G given in Figure 1a is presented in Figure 2. As shown in Figure 2, reflecting graph theory terminology, S_1 is the root augmented node in the AT since subnetwork S_1 is the root subnetwork in Figure 1. S_2 and S_4 are located in the middle of the AT, while S_3, S_5, and S_6 are located at the leaves of the AT.

[24] The AT is now used to illustrate the two novel features of the proposed optimization method, which are (i) the optimization is carried out for each subnetwork separately (rather than for the original full network as a whole) *in a predetermined sequence specified by the* directed links in the AT and (ii) each subnetwork design optimization incorporates the solutions for all the subnetworks that are immediately attached to this subnetwork based on the direction of the directed links in the AT.

[25] Referring the novel feature (i), as specified by the directed links in the AT given in Figure 2, S_3, S_5, and S_6 are first separately optimized, followed by S_4; then S_2 and finally is S_1. That is, subnetwork optimization takes place from the leaves to the root of the AT, which is opposite to the flow direction of the AT (that is from the root to the leaves as the supply source node is included in the root-augmented node).

[26] In order to facilitate the implementation of the novel feature (ii), for each subnetwork represented by an augmented node in the AT, all the other subnetworks that are immediately attached to this subnetwork based on the direction of the directed links are defined as its correlated subnetworks ϕ. Based on this definition, the correlated subnetworks for each subnetwork given in Figure 2 is $\varphi(S_1) = \{S_2\}$, $\varphi(S_2) = \{S_3, S_4\}, \ldots \varphi(S_3) = \varnothing$, $\varphi(S_4) = \{S_5, S_6\}$, $\varphi(S_5) = \varnothing$, and $\varphi(S_6) = \varnothing$. Based on the novel feature (ii) of the proposed method, each subnetwork design optimization needs to include the solutions for all the subnetworks in its φ.

[27] By applying the two novel features to the water network given in Figure 1 (its AT is presented in Figure 2), S_3, S_5, and S_6 should first be individually optimized and they do not consider other networks during optimization since their $\varphi = \varnothing$. Then, S_4 is optimized while incorporating the solutions for S_5 and S_6 since $\varphi(S_4) = \{S_5, S_6\}$. Subsequently, S_2 is optimized and S_3 and S_4 are included during the optimization ($\varphi(S_2) = \{S_3, S_4\}$). Finally, S_1 is optimized and S_2 is included ($\varphi(S_1) = \{S_2\}$).

[28] As previously mentioned, two distinct optimization steps are utilized in the proposed method when dealing

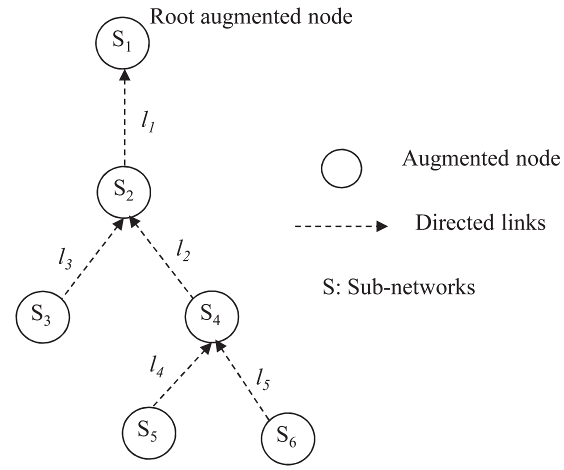

Figure 2. The directed AT of the water network G given in Figure 1a.

Table 1. Nodal and Pipe Information of N_1

Link	Length (m)	Node	Water Demand (L/s)
1	800	*v*	Reservoir
2	750	*a*	25
3	600	*b*	27
4	485	*c*	32
5	452	*d*	15
6	478	*e*	48
7	492	*f*	20
8	562	*g*	124
9	145	*h*	14
10	785	*i*	32
11	456	*j*	13
12	325	*k*	17
13	148	*l*	22
14	478	*m*	42
15	528	*n*	89
16	400	*o*	26
17	258	*p*	23
18	547	*q*	11
19	500	*r*	19
20	200	*s*	17
21	200	*t*	16
22	900	*u*	32
23	654		
24	698		
25	250		
26	700		
27	254		

with the optimization design for a WDN, which are preconditioning optimization for the subnetworks (Step 3) and the final optimization for the subnetworks (Step 4). The details of these two proposed optimization algorithms are discussed in the later section.

[29] The water network given in Figure 1a (denoted as N_1) is used to illustrate the proposed optimization approach. The elevation of all the demand nodes is 10 m, and the head provided by the supply source node (v) is 45 m. The minimum head requirement for each demand node is 35 m. The water demands for each node and the length for each pipe are given in Table 1. The Hazen-Williams coefficient for each new pipe is 130. A total of 14 diameters ranging from 150 to 1000 mm are used for the N_1 design. The pipe diameters and the cost for each diameter are given by *Kadu et al*. [2008].

3.3. Preconditioning Optimization for the Subnetworks (Step 3)

[30] Three typical subnetworks can be defined for the decomposed network in the proposed method, including the subnetworks at the leaves (L(AT)), subnetworks in the middle of the directed augmented tree (M(AT)), and the root subnetwork (Rt(AT)). For the subnetworks represented by augmented nodes in Figure 2, $\{S_3, S_5, S_6\} \in L$(AT), $\{S_2, S_4\} \in M$(AT), and $S_1 \in$Rt(AT).

[31] Subnetworks at the leaves [$S \in L$(AT)] differ from other subnetworks as their $\varphi = \emptyset$. The root subnetwork [$S \in$Rt(AT)] is characterized by its known available head, since it includes the supply source node of the original WDN. The available heads of the subnetworks in the middle of the directed augmented tree ($S \in M$(AT)) are unknown and their $\phi \neq \emptyset$, which are different from $S \in L$(AT) and $S \in$Rt(AT). In the proposed method, the optimization process for each type of subnetwork varies.

3.3.1. Optimization for the Subnetwork at the Leaves of the *AT*

[32] The subnetworks at the leaves ($S \in L$(AT)) are first optimized in the proposed method. Since no supply source node exists for each $S \in L$(AT), each subnetwork *cut node* connecting the $S \in L$(AT) and the $S \in M$(AT) is assumed to be a supply source node for $S \in L$(AT). Therefore, the subnetwork cut nodes f, m, and n represent the supply source nodes for S_3, S_5, and S_6, respectively, as shown in Figure 1b.

[33] Since the available head (H) at a subnetwork cut node is unknown, a series of sequential heads (H) between $H_{\min}$ and $H_{\max}$ are assigned for the subnetwork cut node, where $H_{\min}$ is the maximum value of all minimum required nodal heads across the whole subnetwork that is being optimized and $H_{\max}$ is the allowable head provided by the supply source node of the original network. The logic behind setting the head range [i.e., $H \in (H_{\min}, H_{\max})$] is that no feasible solution can be found if the available head at the subnetwork cut node is smaller than the maximum value of the minimum head constraints at all subnetwork nodes, and the maximum head of the subnetwork cut node cannot be greater than the head of the supply source node. A series of different H, $H \in (H_{\min}, H_{\max})$, with a particular interval (say 1 m) are used for the subnetwork cut node in order to enable subnetwork optimization.

[34] For each value of H assigned to a subnetwork cut node, a differential evolution (DE) algorithm combined with a hydraulic simulation model (EPANET2.0) is used to optimize the subnetwork design, while satisfying the head requirements for each node within the subnetwork. The minimum pressure head excess H_{excess} ($H_{\text{excess}} \geq 0$) across the subnetwork is obtained for each optimal solution associated with a particular value of H at the subnetwork cut node. This indicates that the head at the subnetwork cut node can be further reduced by H_{excess} while maintaining the feasibility of this optimal solution. The head H at the subnetwork cut node is then adjusted to H^*, where $H^* = H - H_{\text{excess}}$, which is the minimum head requirement at the subnetwork cut node for the optimal solution associated with the minimum pressure head excess H_{excess}.

[35] Consequently, a solution choice table (ST) is constituted for the subnetwork that is being optimized by assigning a series of different values of H to its assumed supply source node, subnetwork cut node. In the ST, H^*, optimal solution costs and the subnetwork configurations (pipe diameters) of optimal solutions are included, and each unique H^* is associated with a unique optimal solution (including the cost and the subnetwork configuration).

[36] The subnetwork S_6 in N_1 is used to illustrate the proposed optimization method for the $S \in L$(AT). The $H_{\min}$ and $H_{\max}$ values for S_6 are 35 and 45 m, respectively, where $H_{\min}$ is the maximum head requirement for all nodes across S_6 (35 m) and the $H_{\max}$ is the allowable head provided by the actual supply source node (45 m). A series of H ranging from 35 to 45 m with an increment of 1 m, i.e.,$H = \{36, \ 37, \ 38, \ldots, 45\}$ is used for the subnetwork cut node n to optimize the design for S_6. Note that no feasible solution can be found if $H = 35$ m is assigned to node n as the minimum head requirement for S_6 is 35 m. Thus, the value of $H = 35$ m is not included in the series of

Table 2. Optimal Solutions for S_6 of N_1

H at Subnetwork Cut Node n (m)	Minimum Pressure Head Excess H_{excess} (m)	$H^* = H - H_{excess}$ (m)	Cost of Optimal Solutions ($)	Pipe Diameters for Each Optimal Solution[a] (mm)
36	0.014	35.986	155,487	450, 250, 300, 150, 300
37	0.231	36.769	130,288	400, 200, 300, 150, 250
38	0.157	37.843	115,622	350, 200, 250, 150, 300
39	0.120	38.880	108,175	350, 150, 250, 150, 250
40	0.397	39.603	105,079	350, 150, 250, 150, 200
41	0.513	40.487	98,175	300, 150, 250, 150, 250
42	0.790	41.210	95,079	300, 150, 250, 150, 200
43	0.402	42.598	92,032	300, 150, 200, 150, 200
44	1.402			
45	0.160	44.840	89,168	250, 150, 200, 150, 250

[a]The pipe diameters are for links 23–27 of N_1 network (Figure 1a) from the first to the last pipe, respectively. Note that only one solution is recorded in the table for the identical solutions (having the same H^*, optimal cost, and pipe diameter for links).

H values assigned for the subnetwork cut node n. The optimal solution for each value of H, the minimum pressure head excess (H_{excess}), and the H^* value for each optimal solution for S_6 are given in Table 2.

[38] As can be seen from Table 2, with values of H given at the subnetwork cut node n from the smallest to the largest (the first column of Table 2), values of H^* are also ordered from the smallest to the largest, while its corresponding optimal solution is ordered from the largest to the smallest in terms of cost. This is due to the fact that a lower cost solution is achieved if a higher head is provided at the subnetwork cut node. This solution choice table is denoted as ST_n since the subnetwork cut node n is the assumed supply source for S_6. It is noted that the identical solutions (having the same H^*, optimal cost, and pipe diameters for links) are removed from the solutions choice table. For example, for heads of 43 and 44 m in ST_n, only one solution is left in the solution choice table.

[39] Each $S \in L(AT)$ is optimized using the same approach as for S_6 described above, and hence a solution choice table is constituted for each one after optimization. For N_1 case study, in addition to S_6, S_3 and S_5 are also subnetworks at the leaves of the directed augmented tree (see Figure 2). For S_3 and S_5, $H_{min} = 35$ m and $H_{max} = 45$ m, hence a series of values for $H = 36, \quad 37, \quad 38, \ldots, 45$ are used for the subnetwork cut nodes f and m to optimize the design for the S_3 and S_5, respectively. As previously explained, $H = 35$ m is not assigned to the subnetwork cut nodes as no feasible solution can be found with this assumed head value (the minimum head requirement is 35 m for the N_1 case study).The obtained solution choice tables for S_3 and S_5 are presented in Table 3 (the identical solutions have been removed from solution choice tables).

3.3.2. Optimization for the Subnetwork in the Middle of the *AT*

[41] The optimization for the $S \in M(AT)$ is carried out once the optimization for $S \in L(AT)$ has been finished. For each $S \in M(AT)$, the water demands at each subnetwork cut node have to be increased by the flows in the directed links to this subnetwork that is being optimized (note the direction of the flows is opposite to the directed links). For the example given in Figure 1b, the water demands at subnetwork cut nodes f, m, and n [$f \in S_2$, $\{m, n\} \in S_4$, $\{S_2, S_4\} \in M(AT)$] are increased by the flows in directed link l_3, l_4, and l_5, respectively (see Figure 2), which are actually the demands of subnetworks S_3, S_5, and S_6, respectively. The water demand at subnetwork cut node e is added by the flows in directed link l_2, which are the total demands of subnetwork S_4, S_5, and S_6, as shown in Figure 2. It is noted that each $S \in L(AT)$ is connected to the original entire network via only one subnetwork cut node, while each $S \in M(AT)$ is attached to the whole system with multiple subnetwork cut nodes.

Table 3. Solution Choice Tables for S_3 and S_5 of N_1

Subnetwork	H at Assumed Supply Source Node (m)	$H^* = H - H_{excess}$ (m)	Cost of Optimal Solutions ($)	Pipe Diameters for Each Optimal Solution[a] (mm)
Solution choice table for S_3 [ST(f)]	36	35.845	90,200	500, 150, 350, 200, 200
where node f is the assumed supply	37	36.939	73,900	400, 150, 300, 150, 200
source node for S_3	38	37.765	67,620	400, 150, 250, 150, 200
	39	38.886	63,553	350, 150, 250, 150, 150
	40	39.916	62,915	300, 150, 250, 150, 200
	41	40.903	60,483	400, 150, 200, 150, 150
	42	41.547	57,995	350, 150, 200, 150, 200
	43	42.575	57,357	300, 150, 200, 150, 200
	44, 45	43.054	55,778	300, 150, 200, 150, 150
Solution choice table for S_5 [ST (m)]	36	35.995	74,686	350, 250, 150, 150
where node m is the assumed	37	36.864	64,603	300, 200, 150, 150
supply source node for S_5	38	37.925	62,469	300, 150, 150, 150
	39	38.649	57,717	250, 200, 150, 150
	40, 41, 42, 43, 44	39.710	55,583	250, 150, 150, 150
	45	44.607	51,623	200, 200, 150, 150

[a]The pipe diameters are for links 9–13 of N_1 network (Figure 1a) in S_3 and for links 19–22 of N_1 network in S_5 from the first to the last, respectively.

[42] Among these subnetwork cut nodes attached to each $S \in M$(AT), the one that is located at the upstream end based on the flow direction is assumed as a supply source. Thus, subnetwork cut nodes c and e are the assumed supply sources for S_2 and S_4, respectively, for the water network given in Figure 1. A series of different H, $H \in (H_{\min}, H_{\max})$, with a particular interval (of again say 1 m) are assigned to the subnetwork cut node for optimizing the $S \in M$(AT), which is the same approach as for optimizing $S \in L$(AT) described in section 3.3.1.

[43] It is important to note that for each $S \in M$(AT), at least one subnetwork is located at its immediately adjacent downward side based on the direction of the directed links in the AT, i.e., $\phi \neq \varnothing$. In the proposed method, the optimization of each $S \in M$(AT) needs to include all the subnetworks in its ϕ and the solutions for the subnetworks in its ϕ are selected from their corresponding solution choice tables during optimization. The formulation of the optimization problem for each $S \in M$(AT) is given by

$$\text{Minimize} \quad F' = F(S) + \sum f(\varphi(S)), \qquad S \in M(\text{AT}), \tag{5}$$

[44] Subject to:

$$H_{S,k}^{\min} \leq H_{S,k} \leq H_{S,k}^{\max} \qquad k = 1, \ldots., nsj, \tag{6}$$

$$G(H_{S,k}, D_s) \quad = \quad 0, \tag{7}$$

$$f(\varphi(S)) \in ST(\varphi(S)), \tag{8}$$

where F' is the total cost (to be optimized); $F(S)$ is the cost of the subnetwork S ($S \in M$(AT)); $\varphi(S)$ is all subnetworks in the φ of S (the φ is defined in section 3.2); $\sum f(\varphi(S))$ is total costs for all other subnetworks in the φ; $G(H_{S,k}, D_s)$ is the nodal mass balance and loop (path) energy balance equations for the subnetwork S, which is handled by a hydraulic simulation package (EPANET2.0 in this study); $H_{S,k}$ is the nodal head of the node $k = 1, \ldots, nsj$; nsj is the number of nodes within the subnetwork S; $H_{S,k}^{\min}$ and $H_{S,k}^{\max}$ are the lower and upper head boundaries at the nodes of S; and ST $(\varphi(S))$ is the solution choice tables of subnetworks in the φ.

[45] As shown from equations (5)–(8), although the total costs of the $S \in M$(AT) and all subnetworks in its φ are to minimized, only the cost and nodal head constraints of the $S \in M$(AT) are explicitly handled by an optimization algorithm (DE used in this study). This is because the optimal solutions for the subnetworks in the φ [denoted as $f(\varphi(S))$] are selected from their corresponding solution choice tables $ST(\varphi(S))$ during optimization (equation (8)). In addition, head constraints of subnetworks in the φ are also handled by their corresponding solution choice tables. This is one of the novel aspects of the proposed optimization method. The details of the proposed method in terms of selecting optimal solutions from solutions choice tables and handling constraints during the optimization for the $S \in M$(AT) are given as follows.

[46] The optimization of S_4 in N_1 is used to illustrate the proposed methods for optimizing the $S \in M$(AT). For the water network given in Figure 1 and its AT shown in Figure 2, $\varphi(S_4) = \{S_5, \quad S_6\}$, and hence S_5 and S_6 are included when S_4 is optimized. For S_4 optimization, different values of $H = 36, \quad 37, \quad 38, \ldots, 45$ are used for the assumed supply source e ($H_{\min} = 35$ m and $H_{\max} = 45$ m) and then a DE is employed to optimize the design for S_4 for each H value.

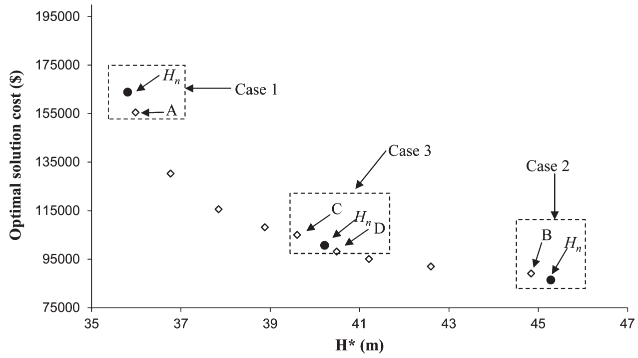

Figure 3. H^* versus the optimal solution cost for S_6 of N_1 (solution selection).

[47] The total cost, including the cost of S_5, the cost of S_6, and the cost of S_4 is to be minimized for the DE applied to optimize S_4 [$\varphi(S_4) = \{S_5, \quad S_6\}$]. For each individual solution in the DE algorithm, the head at the subnetwork cut nodes m (H_m) and n (H_n) are tracked after the hydraulic simulation for S_4 (EPANET2.0). Then the optimal solution for S_5 and S_6 are selected from their corresponding solution choice tables ST_m and ST_n based on assigning H_m and H_n to the subnetwork cut nodes m and n. As H_m and H_n may not precisely equal any particular H^* values in ST_m and ST_n, an approach is proposed in this study to select the appropriate optimal solutions based on the values of H_m and H_n. Figure 3 illustrates the details of this selection approach, and the values of H^* versus the optimal solution costs in the solution choice table ST_n for S_6 is presented in Figure 3 to facilitate the explanation.

[48] For each individual solution of the DE applied to optimize S_4, H_n (head at the subnetwork cut node n) is obtained after hydraulic simulation for S_4. Based on the value of H_n, three cases exist for selecting the optimal solution for S_6, as shown in Figure 3:

[49] Case 1: If H_n is smaller than the minimum H^* [$H^*(A)$] in ST_n, the cost associated with the minimum H^* (the cost of solution A in Figure 3) is added to the total cost of this individual solution and the network configuration (pipe diameters) associated with [$H^*(A)$] is assigned for S_6. In addition, a penalty is applied to this individual solution as no feasible solution is found for S_6.

[50] Case 2: If H_n is greater than the maximum H^* ($H^*(B)$) in ST_n, the cost associated with the maximum H^* (the cost of solution B in Figure 3) is added to the total cost of this solution and the network configuration (pipe diameters) associated with [$H^*(B)$] is assigned for S_6.

[51] Case 3: If H_n is between two adjacent H^* values in ST_n, the solution has the H^* immediately smaller than the H_n is selected and its cost is added to the total cost of this individual solution. As shown in Figure 3, the solution C will be selected for S_6 if the individual solution has a H_n between $H^*(C)$ and $H^*(D)$, resulting in a pressure head excess of $H_n - H^*(C)$ for S_6. As such, the solution

Table 4. Solution Choice Table for S_4 of N_1

H at Subnetwork Cut Node e (m)	$H^* = H - H_{excess}$ (m)	Cost of Optimal Solutions ($)	Pipe Diameters for Each Optimal Solution[a] (mm) in the Solution Choice Table for S_4 [ST(e)]		
			S_4	S_5	S_6
36	–	Infeasible	–	–	–
37	36.938	542,915	700, 600, 450, 600, 150	350, 250, 150, 150	450, 250, 300, 150, 300
38	37.936	484,396	600, 500, 400, 600, 150	350, 250, 150, 150	450, 250, 300, 150, 300
39	38.916	437,211	600, 500, 400, 500, 150	300, 200, 150, 150	400, 200, 300, 150, 250
40	39.752	414,439	600, 450, 350, 450, 150	300, 200, 150, 150	400, 200, 300, 150, 250
41	40.939	392,887	600, 450, 350, 450, 150	250, 200, 150, 150	350, 200, 250, 150, 300
42	41.860	380,809	500, 450, 350, 500, 150	300, 150, 150, 150	350, 200, 250, 150, 300
43	42.974	368,869	500, 500, 350, 400, 150	250, 200, 150, 150	350, 150, 250, 150, 250
44	43.783	348,862	500, 400, 300, 400, 150	250, 200, 150, 150	350, 200, 250, 150, 300
45	44.844	339,281	500, 400, 300, 400, 150	250, 150, 150, 150	350, 150, 250, 150, 250

[a]The pipe diameters are for links 14–27 of N_1 network from the first to the last, respectively (see Figure 1a).

selected from ST_n can be guaranteed to be feasible as the solution with H^* smaller than H_n is chosen. The network configuration (pipe diameters) associated with $[H^*(C)]$ is assigned for S_6 in this case.

[52] The approach described above is also used to include the cost of S_5 when a DE is used to optimize S_4. As such, although only the pipes in S_4 are handled by the DE, the solutions in the DE actually include the total cost of S_4, S_5, and S_6. Once the DE has converged to the final optimal solution for S_4, the minimum pressure head excess H_{excess} for this optimal solution is determined by

$$H_{excess} = \min[H^*_{excess}, \quad (H_m - H^*(ST_m), \quad (H_n - H^*(ST_n)], \tag{9}$$

where H^*_{excess} is the minimum pressure head excess across all the demand nodes for S_4 that is being optimized; $H^*(ST_m)$ and $H^*(ST_n)$ are the values of H^* associated with the solutions selected for S_5 and S_6 from ST_m and ST_n, respectively, based on the approach illustrated in Figure 3. The head H at the subnetwork cut node e is then adjusted to H^*, where $H^* = H - H_{excess}$.

[53] For each different value of H assigned to the subnetwork cut node e, the optimal cost solution for S_4, S_5, and S_6 is obtained by the DE algorithm. In addition, the minimum pressure head excess H_{excess} is obtained using equation (9), and hence the value of H^* ($H^* = H - H_{excess}$) is obtained for each optimal solution. As such, a solution choice table for S_4 is formed, in which, H^*, the optimal solution cost and subnetworks configuration (pipe diameters for S_4, S_5, and S_6) of the optimal solution are included, which is presented in Table 4.

[55] As shown in Table 4, a total of nine different feasible optimal solutions were found by the DE applied to S_4 optimization with the heads at the assumed source node e being $36, \quad 37, \quad 38, \ldots, 45$. No feasible solution was found with $H = 36$ m assigned to node e. In the solution choice table ST(e) for S_4, the values of H^* across the subnetworks of S_4, S_5, and S_6, the total cost of S_4, S_5, and S_6, the design for each of these three subnetworks are included.

[56] As shown in Figure 2, $\varphi(S_2) = \{S_3, \quad S_4\}$, thus S_3 and S_4 are included when S_2 is optimized in the proposed method. The subnetwork S_4 is optimized before S_2 as the optimization sequence in the proposed method is from the leaves to the root based on the directed augmented tree. The approach described in Figure 3 was used to select the solutions for S_3 and S_4 from their corresponding solution choice tables when S_2 is optimized. A similar method presented in equation (9) was utilized to obtain the H_{excess} for each optimal solution of S_2.

[57] Since $H_{min} = 35$m and $H_{max} = 45$ m for S_2, $H = 36, \quad 37, \quad 38, \ldots, 45$ were used for the assumed supply source node c to optimize S_2. In a similar way to that for S_4, a solution choice table is formed for S_2 after optimization, which is denoted as ST(c) as the subnetwork cut node c is the assumed supply source node. The final solutions in the ST(c) are the optimal solutions for S_2, S_3, and S_4, which is actually the total optimal solutions for S_2, S_3, S_4, S_5, and S_6 as the solutions in S_4 have already included S_5 and S_6. The designs for the optimal solutions of S_2, S_3, S_4, S_5, and S_6 are also included in the ST(c).

[58] The formulation of the optimization problem given from equations (5)–(8) and the approach used for S_4 optimization (Figure 3 and equation (9)) are employed to optimize each $S \in M$(AT), thereby a solution choice table is constituted for each subnetwork in the middle of the directed augmented tree AT.

3.3.3. Optimization for the Subnetwork at the Root of the *AT*

[59] The root subnetwork is the final one to be optimized in the proposed method. As the supply source node in the original full WDN is included in $S \in$Rt(AT), the available head is known when optimizing $S \in$Rt(AT). For the $S \in$Rt(AT), $\varphi \neq \emptyset$ and hence the approach used for the optimization of $S \in M$(AT) is also employed to deal with the optimization of the subnetwork at the root of the AT. For the example given in Figure 1, $S_1 \in$Rt(AT) and $\varphi(S_1) = S_2$, thus ST(c) is used to provide the optimal solution for S_2 when S_1 is optimized.

[60] An approximate optimal solution with a cost of $1.021 million is obtained after S_1 optimization, which is also the optimal solution for the whole N_1 network. This is because S_5 and S_6 were included when S_4 was optimized, S_3 and S_4 were included when S_2 was optimized, and S_2 was in turn included when S_1 was optimized in the proposed method. Thus, the final optimal solutions from the optimization of S_1 are the optimization results for the original full network N_1.

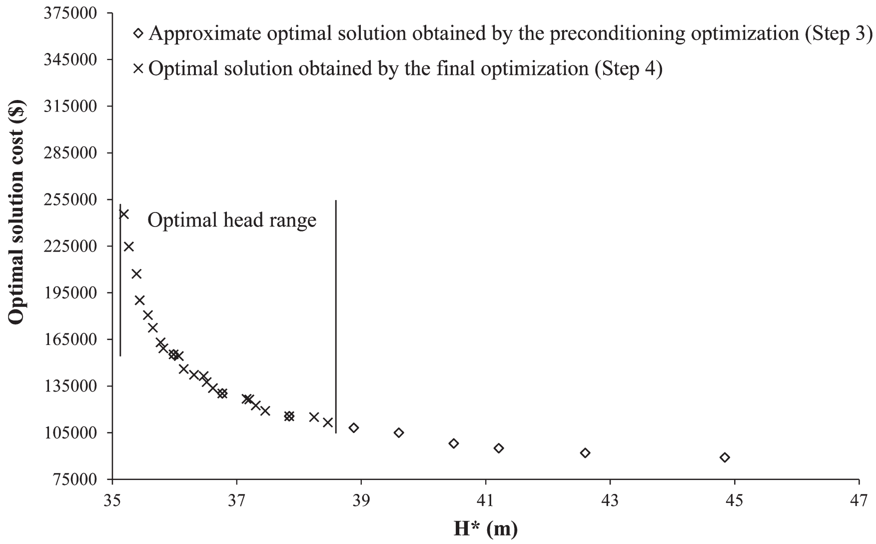

Figure 4. H^* versus the optimal solution cost for S_6 of N_1.

[61] During the preconditioning optimization for the subnetworks in the proposed method, a series of H with a relatively larger interval ($H \in (H_{\min}, H_{\max})$) is used for the subnetwork cut nodes (1 m in this study). This aims to approximately explore the search space of the original full network, thereby producing an approximate optimal solution. This approximate optimal solution is used to specify promising regions for the entire search space, allowing the next step (Step 4) of the final optimization for the subnetworks to be conducted. The final optimization for the subnetworks method is described in the next section.

3.4. Final Optimization of the Subnetworks (Step 4)

[62] Based on the approximate optimal solution obtained by the preconditioning subnetwork optimization, an optimal head (H) for each subnetwork cut node can be determined. An optimal head range $\Re(H)$ is created for each subnetwork cut node through expansion of the obtained optimal head, i.e., $\Re(H) = [H - \delta,\ H + \delta]$. In this proposed method, $\delta = 2$ m is used to obtain the optimal head range $\Re(H)$.

[63] During the final optimization of the subnetworks, all the subnetworks are optimized employing the same approach used for preconditioning optimization for subnetworks, while the head assigned for the subnetwork cut nodes is varied. For the preconditioning optimization for subnetworks, a whole range of possible H values between $H_{\min}$ and $H_{\max}$ at the subnetwork cut nodes with a relatively large increment (1 m) was used, while a series of H values within the optimal head range $\Re(H)$ with a relatively small increment (e.g., 0.1 m) was used for subnetwork cut nodes during the final optimization of subnetworks. The optimization sequence is also taken from the leaves to the root specified by the directed augmented tree in the final optimization step. The solution choice table for each subnetwork created after the preconditioning optimization is updated during the final optimization step.

[64] For the example given in Figure 1, the heads at the subnetwork cut nodes n is 36.8 m based on the approximately optimal solution obtained after the preconditioning subnetworks optimization ($1.021 million). Thus, the optimal heads range for node n is $\Re(H_n) = [34.8,\quad 38.8]$. The H^* versus the optimal solution cost for S_6 using the head given by the obtained optimal head range $\Re(H)$ with an increment of 0.1 m is given in Figure 4.

[65] A total of 23 different optimal solutions were found for S_6 of the N_1 case study with the head given at node n within the optimal head range $\Re(H)$, compared to only nine different approximate optimal solutions generated during the preconditioning optimization step for S_6. This shows that the proposed final optimization method is able to further exploit the promising regions specified by the optimal head range in the preconditioning phase, thereby allowing more optimal solutions to be located. This is also shown by Figure 4 that a number of additional optimal solutions were found by the final subnetwork optimization process between two adjacent optimal solutions found initially by preconditioning.

[66] All other subnetworks of N_1 are optimized based on the obtained optimal head range for each subnetwork cut node during the final optimization step. The final optimal solution for the N_1 case study obtained after the final optimization step was $1.016 million, a value lower than the optimal solution generated by the preconditioning optimization for subnetworks (Step 3) with a cost of $1.021 million. This shows that the proposed final optimization of the subnetworks approach is effective in improving the quality of optimal solutions generated by the preconditioning optimization step.

3.5. Summary of the Proposed Method

[67] The proposed method does not need to know the actual head constraints at the subnetwork cut nodes, instead a series of assumed heads are assigned at subnetwork cut nodes. Then the DE optimization is used to seek the least-cost design of the subnetwork for each assumed head at the subnetwork cut node, while satisfying the specified head requirement at each node (such as 35 m for the N_1

Table 5. EA Parameter Values and the Hydraulic Simulation Time for Each Subnetwork and the Full N_1

EAs	Network	Number of Decision Variables and the Search Space Size	Population Size (N)	Maximum Number of Allowable Evaluations	Computational Time for 1000 Simulations[a] (s)
SDE	N_1	27 (8.82×10^{30})	100	500,000	0.765
GA	N_1	27 (8.82×10^{30})	200	800,000	0.765
DE used in the proposed method	S_1	5 (537,824)	20	2,000	0.105
	S_2	3 (2,744)	20	2,000	0.081
	S_3	5 (537,824)	20	2,000	0.110
	S_4	5 (537,824)	20	2,000	0.108
	S_5	4 (38,416)	20	2,000	0.095
	S_6	5 (537,824)	20	2,000	0.098

[a]The 1000 simulations were based on randomly selected network configuration and conducted on the same computer configuration (Pentium PC (Inter R) at 3.0 GHz).

network). This results in the development of a solution choice table for each subnetwork (except the root subnetwork). For each solution choice table, every H^* is associated with an optimally feasible solution (determined by EPANET2.0) for its corresponding subnetwork. Therefore, the final optimal solutions can be guaranteed to be feasible for the whole original WDN since all the selected optimal solutions from solution choice tables are feasible (i.e., all the head constraints are satisfied).

[68] The proposed method recognizes the fact that, although decomposed, subnetworks in a WDN are in reality always interconnected and never truly independent of one another. Thus, for each subnetwork optimization, all the subnetworks in its φ are considered. Therefore, the optimal solution obtained for each subnetwork is actually the optimal solution as a whole of this subnetwork and all the subnetworks in its φ. As the optimization is carried out from the leaves to the root along the assigned directed links in the directed augmented tree, the root subnetwork contains all the subnetwork optimization results by use of solution choice tables. Consequently, the optimal solution for the root subnetwork is actually the final solution for the whole WDN.

[69] In the proposed method, each subnetwork optimization also considers all the subnetworks in its φ, while the number of decision variables handled is the number of pipes of the subnetwork that is currently being optimized plus the number of solution choice tables that are associated with the subnetworks in the φ. This is because all the optimal solutions for the subnetworks in the φ are already provided by their corresponding solution choice tables.

4. Case Study Results and Discussion

[70] A total of five case studies are used to verify the effectiveness of the proposed optimization approach, including one artificial water network, two benchmark case studies, and two real-world water networks. A DE combined with a hydraulic solver (EPANET2.0) was employed to optimize each subnetwork design. In addition to the proposed graph decomposition optimization approach, a standard DE algorithm (SDE) and a GA with tuned parameters were applied to each case study in order to enable a performance comparison with the proposed method. The SDE algorithm used in this paper was given in *Zheng et al*. [2013b] and the GA algorithm with integer coding, constraint tournament selection, and an elite strategy described in *Zheng et al*. [2013c] was used in this study.

4.1. Case Study 1: Artificial Network 1 (N_1) (27 Decision Variables)

[71] The layout and the network details of artificial network 1 (N_1) were previously provided in Figure 1a and Table 1 as examples of network decomposition. The decomposition results (subnetworks S_1–S_6) and the directed augmented tree of N_1 (directed links l_1–l_5) are provided in Figures 1b and 2, respectively. Table 5 summarizes the DE parameter values used for optimizing the full N_1 and each network into which it has been decomposed by the graph theory algorithm. In addition, the computational times for running simulation on the whole N_1 and each subnetwork (S_1–S_6) are provided. A mutation weighting factor (F) of 0.5 and a crossover rate (CR) of 0.5 were selected based on the results of a few parameter trials for the DE used in the proposed method, while the parameters of the SDE and GA have been fine-tuned through extensive parameter calibration. The best parameter values obtained were $F = 0.6$, $\mathrm{CR} = 0.7$ for the SDE, and crossover probability (P_c) with 0.9 and mutation probability (P_m) with 0.03 were selected for the GA.

[73] As previously mentioned, a total of 14 discrete diameters can be used for the N_1 case study, thus the total search space size is $14^{27} \approx 8.82\times10^{30}$. The search spaces for subnetworks are significantly reduced compared to the original whole network as shown in Table 5, Hence, the population size (N) and maximum number of allowable evaluations assigned for the subnetwork optimization are considerably less than those used by the original full network optimization as shown in Table 5.

[74] The results of the proposed method and SDE applied to the N_1 case study are provided in Table 6. As shown in Table 6, the current best solution for the N_1 case study is \$1.016 million. This solution was found by the proposed method after the final optimization step with a success rate of 100% based on 50 different runs using different random number seeds, compared to 90% returned by the SDE. The best solution found by the GA was \$1.019 million, which is 0.3% higher than the current best solution (\$1.016 million) for this case study. In terms of average cost of solutions based on 50 runs, the proposed method exhibits similar performance with the SDE but significantly outperformed the GA.

[79] In order to enable a fair comparison in terms of efficiency, all the computational times required by the proposed method has been converted to an equivalent number of full N_1 evaluations using the same computer

Table 6. Algorithm Performance for the N_1 Case Study

Algorithm	Number of Different Runs	Best Solution Found in Millions ($)	Trials With Best Solution Found (%)	Average Cost Solution in Millions ($)	Average Number of Equivalent Evaluations to Find the Best Solution
Proposed method[a]	50	1.021	0	1.021	15,608[b]
Proposed method[c]	50	1.016	100	1.016	78,039[b]
SDE[d]	50	1.016	90	1.017	152,854
GA[d]	50	1.019	0	1.027	392,676

[a]The results of the proposed method after preconditioning subnetwork optimization (Step 3).
[b]The results of the proposed method after final subnetwork optimization (Step 4).
[c]The total computational overhead required by the proposed method has been converted to an equivalent number of whole network (N_1) evaluations.
[d]Parameters were tuned.

configuration. These include the computational time used for identifying the subnetworks (equivalent to nine full N_1 evaluations) and the computational time spent for the subnetworks optimization (Steps 3 and 4). This conversion was made for each case study to allow an efficiency comparison between the proposed method and the SDE. As shown in Table 6, the proposed method required an average number of full N_1 evaluations of 78,039 to find the best solutions after the final optimization step.

[80] The most noticeable advantage of the proposed graph decomposition optimization method is the significantly improved efficiency for finding the current best known solutions compared to the SDE and GA. The proposed method only required an average of 78,039 equivalent full network evaluations to find the optimal solutions, which is only 51% and 20% of those used by the SDE and GA, respectively.

[81] The results of the proposed method after the preconditioning optimization for the subnetworks optimization (Step 3) are also included in Table 6. An approximate solution with a cost of $1.021 million was consistently located by the proposed method after the preconditioning optimization step, which is only 0.5% higher than the current best solution ($1.016 million). However, this approximate solution was found only using 15,608 equivalent full N_1 evaluations, which is only 10% of that required by the SDE. This shows that the proposed preconditioning optimization for the subnetworks (Step 3) is effective as it is able to specify promising regions for the final optimization of the subnetworks (Step 4) with great efficiency.

4.2. Case Studies 2 and 3: Benchmark Case Studies (N_2 and N_3)

[82] Two benchmark case studies, including the New York Tunnel problem (NYTP: N_2) and the Hanoi problem (HP: N_3) have been used to demonstrate the effectiveness of the proposed method. The details of NYTP and HP case studies, including the head constraints, pipe costs, and water demands are given by *Dandy et al*. [1996] and *Fujiwara and Khang* [1990], respectively. For the NYTP and HP case studies, the trees are viewed to be the subnetworks since the blocks are not applicable and the nodes connecting the trees with the other components of the network are viewed as subnetwork cut nodes. The subnetworks and the directed augmented tree for the NYTP and HP case study are presented in Figures 5 and 6, respectively (the original NYTP and HP networks can be found in *Zheng et al*. [2011]).

[83] For the NYTP case study, the optimization sequence for subnetworks is indicated by the directed augmented tree in Figure 5, with the S_2 and S_3 being optimized first, followed by the root subnetwork S_1 ($\varphi(S_1) = \{S_2, S_3\}$). A series of heads with an interval of one foot were used for the subnetwork cut nodes 9 and 12 during the preconditioning optimization for the subnetworks S_2 and S_3 ($H = 272, 273, 274, \ldots, 300$ feet for S_2 and $H = 255, 256, 257, \ldots, 300$ feet for S_3). The DE parameters used for the proposed method and computational simulation time for each subnetwork are given in Table 7.

[85] The optimization results of the proposed graph decomposition optimization method are presented in Table 8. The previously published results for this case study are also included in Table 8 to enable a performance comparison with the proposed method. The current best known solution for the NYTP case study is $38.64 million [*Maier et al*., 2003], and this best solution was found by the proposed method after the preconditioning optimization step (Step 3) with a success rate of 100% based on 100 runs starting with

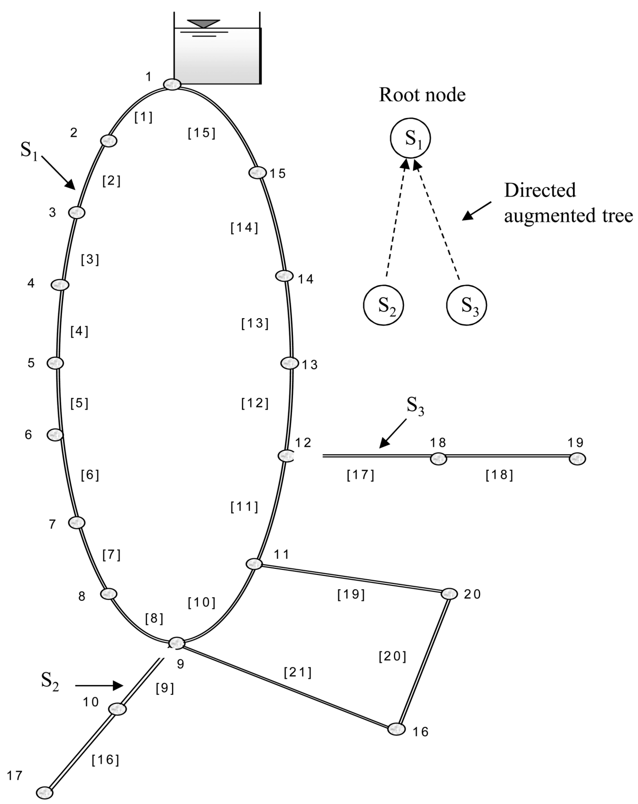

Figure 5. The full network, subnetworks, and the directed augmented tree of the NYTP (N_2) network.

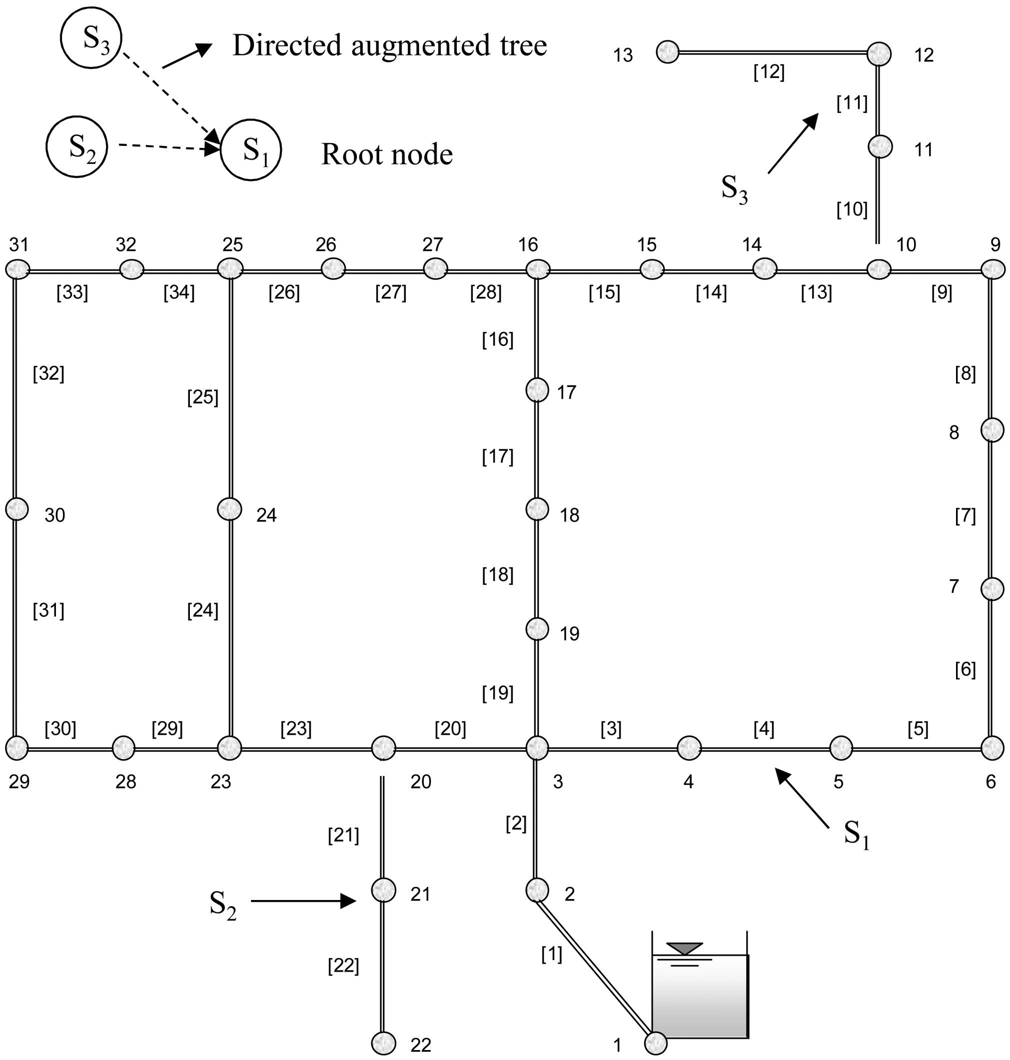

Figure 6. The full network, subnetworks, and the directed augmented tree of the Hanoi (HP: N_3) network.

different random number seeds. The total computational overhead required by the proposed method has been converted to an equivalent number of full NYTP evaluations to enable the efficiency performance with other algorithms. The proposed method exhibits the best performance in terms of percent of trials with the best solution found and the efficiency for the NYTP case study, as can be seen from Table 8. Based on 100 runs, the proposed method only required an average of 3772 equivalent full network evaluations to find the current best known solution, which is significantly lower than those used by other methods shown in Table 8.

[94] The optimization sequence for subnetworks of the HP case study is shown in the directed augmented tree in Figure 6. Subnetworks S_2 and S_3 are optimized first and then the root subnetwork S_1 ($\varphi(S_1) = \{S_2, S_3\}$) is optimized

Table 7. DE Parameter Values for Each Subnetwork of the NYTP and HP Case Studies

Case Study	Network	Number of Decision Variables and the Search Space Size	DE Parameter Values	Maximum Number of Allowable Evaluations	The Computational Time for 1000 Simulations[a] (s)
NYTP	Full network	17 (1.94×10^{25})			0.95
	S_1	17 (2.95×10^{20})	$N = 50$, $F = \text{CR} = 0.5$	10,000	0.810
	S_2	2 (256)	$N = 10$, $F = \text{CR} = 0.5$	1,000	0.100
	S_3	2 (256)	$N = 10$, $F = \text{CR} = 0.5$	1,000	0.110
HP	Full network	34 (2.86×10^{26})			1.156
	S_1	29 (3.68×10^{22})	$N = 80$, $F = 0.7$, $\text{CR} = 0.8$	50,000	0.908
	S_2	2 (36)	$N = 10$, $F = \text{CR} = 0.5$	1,000	0.140
	S_3	3 (216)	$N = 10$, $F = \text{CR} = 0.5$	1,000	0.141

[a]The 1000 simulations were based on randomly selected network configuration and conducted on the same computer configuration (Pentium PC (Inter R) at 3.0 GHz).

Table 8. Summary of the Results of the Proposed Method and Other Algorithms Applied to the NYTP (N_2) Case Study

Algorithm	Number of Runs	Best Solution in Millions ($)	Trials With Best Solution Found (%)	Average Cost in Millions ($)	Average Evaluations to Find First Occurrence of the Best Solution
The proposed method[a]	100	38.64	100	38.64	3,772[b]
NLP-DE[c]	100	38.64	99	38.64	8,277
GHEST[d]	60	38.64	92	38.64	11,464
HD-DDS[e]	50	38.64	86	38.64	47,000
Suribabu DE[f]	300	38.64	71	NA	5,492
Scatter search[h]	100	38.64	65	NA	57,583
GA[g]	100	38.64	45	39.25	54,789

[a]The results of the proposed graph decomposition optimization method after preconditioning subnetworks optimization (Step 3).
[b]The total computational overhead required by the proposed method has been converted to an equivalent number of full NYTP evaluations using the simulation time presented in Table 7.
[c]*Zheng et al*. [2011].
[d]*Bolognesi et al*. [2010].
[e]*Tolson et al*. [2009].
[f]*Suribabu* [2010].
[g]*Zheng et al*. [2012].
[h]*Lin et al*. [2007].

while incorporating the optimal solutions for S_2 and S_3. A series of heads in the range of [30, 100] m with an interval of 1 m were used for the subnetwork cut nodes 20 and 10 during the preconditioning optimization for S_2 and S_3. The DE parameter values for the proposed method applied to subnetworks of the HP case study and the computational simulation time for each subnetwork are shown in Table 7. Table 9 presents the optimization results of the proposed method applied to the HP case study and also the results obtained by previously published algorithms.

[103] The current best known solution for the HP case study was first reported by *Reca and Martínez* [2006], with a cost of $6.081 million. Similarly as for the NYTP case study, the proposed graph decomposition optimization method found the current best known solution for the HP case study after the preconditioning optimization step (Step 3). As can be seen from Table 9, the proposed method was able to locate the current best known solution for the HP case study 98% of the time based on 100 trials, which is higher than all the other algorithms presented in Table 9. In terms of efficiency, the proposed method also performed the best as it found the optimal solutions with an average of 26,540 equivalent full network evaluations, which is fewer than other algorithms in Table 9.

[104] Based on the results of two benchmark case studies (the NYTP (N_2) and HP (N_3)), it can be concluded that the proposed method produced the current best known performance in terms of both the solution quality and efficiency.

4.3. Case Study 4: Network 4 (N_4) (237 Decision Variables)

[105] Network four (N_4) was taken from a town in the southeast of China. N_4 has 237 pipes, one reservoir, and 192 demand nodes. The head provided by the reservoir is 65 m. The minimum pressure requirement for each demand node is 18 m. The Hazen-Williams coefficient for each pipe is 130. A total of 14 pipes ranging from 150 to 1000 mm are used for this network design and the cost of each diameter was provided by *Kadu et al*. [2008]. The original network layout of N_4 is given in Figure 7, and the subnetworks and the directed augmented tree obtained by the proposed decomposition method are presented in Figure 8. As shown in Figure 8, seven subnetworks were identified by the proposed method. The optimization process has to be taken based on the direction from the leaves to the root of the directed augmented tree (Figure 8b).

Table 9. Summary of the Results of the Proposed Method and Other Algorithms Applied to the HP (N_3) Case Study

Algorithm	Number of Runs	Best Solution in Millions ($)	Trials With Best Solution Found (%)	Average Cost in Millions ($)	Average Evaluations to Find First Occurrence of the Best Solution
The proposed method[a]	100	6.081	98	6.081	26,540[b]
NLP-DE[c]	100	6.081	97	6.082	34,609
Suribabu DE[d]	300	6.081	80	NA	48,724
Scatter search[e]	100	6.081	64	NA	43,149
GHEST[f]	60	6.081	38	6.175	50,134
HD-DDS[g]	50	6.081	8	6.252	100,000
GA[h]	100	6.112	0	6.287	384,942

[a]The results of the proposed graph decomposition optimization method after preconditioning subnetworks optimization (Step 3).
[b]The total computational overhead required by proposed method has been converted to an equivalent number of full HP evaluations using the simulation time presented in Table 7.
[c]*Zheng et al*. [2011].
[d]*Suribabu* [2010].
[e]*Lin et al*. [2007].
[f]*Bolognesi et al*. [2010].
[g]*Tolson et al*. [2009].
[h]*Zheng et al*. [2012].

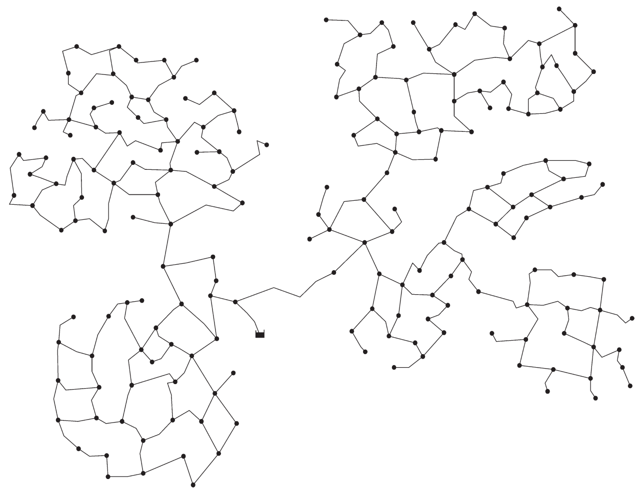

Figure 7. The original full network of N_4 case study.

[106] Table 10 presents the sizes of the networks (including the full network and subnetworks), the population sizes of the DE and GA and the computational time for simulating each network. Values of $F = 0.3$ and $\mathrm{CR} = 0.7$ were selected for the SDE, and values $P_c = 0.9$ and $P_m = 0.005$ were selected for the GA based on an extensive parameter calibration phase. Values of $F = 0.3$ and $\mathrm{CR} = 0.5$ were used for the DE applied to each subnetwork in the proposed graph decomposition optimization method based on a preliminary parameter analysis. It is interesting to note from Table 10 that the total computational running time for hydraulically simulating each subnetwork 1000 times is 8.75 s, which is only 31% of that required by 1000 original full network simulation.

[108] The search space sizes for the original N_4 case study and each subnetwork are included in Table 10. The original search space size for the whole network is $14^{237} \approx 4.29 \times 10^{271}$, while the search space for each subnetwork is significantly reduced. Thus, the DE optimization for the subnetwork requires a lesser number of population size (N) and the maximum number of allowable evaluations compared to the optimization for the original full N_4 network.

[109] Ten different runs with different starting random number seeds were performed for the proposed method and the SDE applied to N_4 case study. The solutions are presented in Figure 9 and the statistical results of these solutions are given in Table 11. It should be noted that the number of evaluations given in Figure 9 for the proposed method is an equivalent number of full N_4 evaluations that was converted by the total computational running time of the proposed method. The computational time used for identifying the seven subnetworks is equivalent to 178 full N_4 evaluations.

[114] As shown in Figure 9, the proposed method is able to find significantly better solutions than the SDE and GA after the final subnetwork optimization (Step 4) with fewer number of equivalent evaluations. In addition, the optimal solutions produced by the proposed method are less scattered than those found by the SDE in terms of distribution. This implies that the proposed method was capable of consistently locating extremely similar or the same final optimal solutions with different starting random number seeds. The optimal solutions found by the proposed method after preconditioning optimization for the subnetworks (Step 3) were higher than those yielded by the SDE and the GA as displayed in Figure 9.

[115] As can be seen from Table 11, the proposed method after the final optimization of the subnetworks (Step 4) found the current best solution for N_3 case study with a cost of \$11.37 million, which is 0.7% and 4.2% lower than the best solutions yielded by the SDE and GA, respectively. The current best solution was found three times out of a total of 10 different runs by the proposed method after Step 4. The average cost solution generated by the proposed method after Step 4 was \$11.38 million, which is only 0.09% higher than the current best solution while 1.2% and 5.4% lower than the average cost solutions of the SDE and GA.

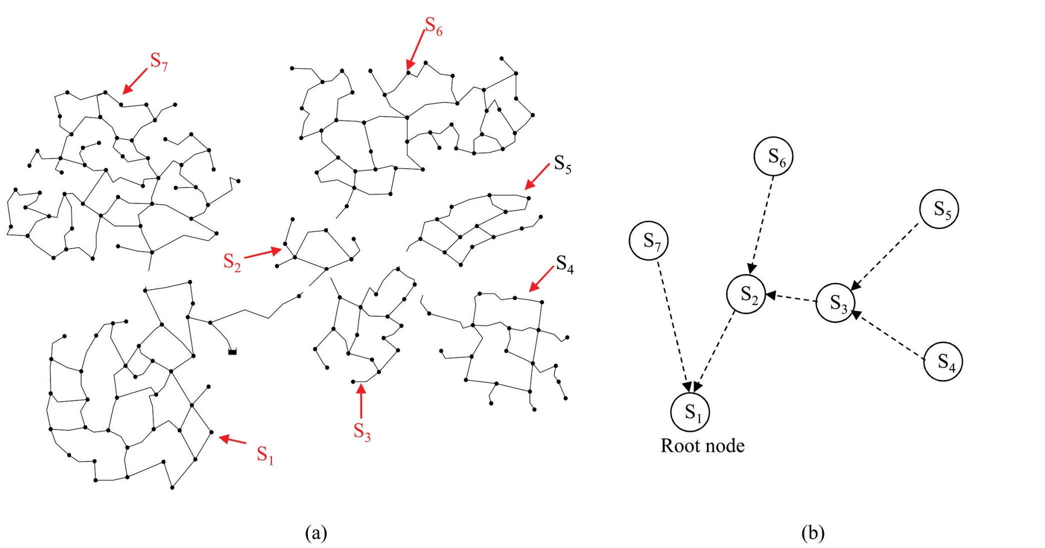

Figure 8. The subnetworks and the directed augmented tree of N_4. (a) The subnetworks. (b) The directed augmented tree.

Table 10. EA Parameter Values and the Hydraulic Simulation Time for Each Subnetwork and the Full N_4

EAs	Network	Number of Decision Variables and the Search Space Size	Population Size (N)	Maximum Number of Allowable Evaluations	The Computational Time for 1000 Simulations[a] (s)
SDE	N_4	237 (4.29×10^{271})	500	5,000,000	28.20
GA	N_4	237 (4.29×10^{271})	500	5,000,000	28.20
DE used in the proposed method	S_1	51 (2.83×10^{58})	100	50,000	2.19
	S_2	9 (2.07×10^{10})	50	5,000	0.32
	S_3	21 (1.17×10^{24})	100	50,000	0.62
	S_4	23 (2.30×10^{26})	100	50,000	0.78
	S_5	18 (4.27×10^{20})	50	25,000	0.62
	S_6	52 (3.97×10^{59})	200	400,000	2.19
	S_7	63 (1.61×10^{72})	200	400,000	2.03

[a]The 1000 simulations were based on randomly selected network configuration and conducted on the same computer configuration (Pentium PC (Inter R) at 3.0 GHz).

[116] In terms of the average number of equivalent evaluations, the proposed method after the preconditioning subnetwork optimization (Step 3) required only 26% of that used by the SDE. Although the solutions found by the proposed method after Step 3 were slightly worse than those located by the SDE and GA, they quickly provided promising regions to allow the further exploitation by the final optimization step (Step 4). After the final subnetwork optimization of the proposed method (Step 4), the solution quality was substantially improved, and the efficiency was still significantly better than the SDE and GA as shown in Table 11.

4.4. Case Study 5: Network 5 (N_5) (433 Decision Variables)

[117] A network (N_5) having 433 pipes and 387 demand nodes has been used in order to verify the effectiveness of the proposed method in terms of dealing with more large and complex networks. The network topology of N_5 was taken from Battle of the Water Networks II (BWN-II) presented in Water Distribution Systems Analysis Conference 2012. The pumps and valves in the original BWN-II network have been replaced by pipes as the aim of this paper is to demonstrate the utility of the proposed method in terms of optimizing the design for the pipes-only network. In addition, the seven tanks in the original BWN-II network have been removed as the proposed method in this paper is currently unable to handle multiple tanks. For this network, the head provided by the reservoir is 75 m, and the minimum pressure requirement for each demand node is 25 m. The Hazen-Williams coefficient for each pipe is assumed to be 130. As the same for case study N_4, 14 pipe choices are used for this network design. The layout of the original N_4 is given in Figure 10, and the decomposed subnetworks and the directed AT are presented in Figure 11.

[118] A total of 12 subnetworks were identified using the proposed method for the N_5 network as shown in Figure 11a. The optimization sequence for the 12 subnetworks is indicated by the directed augmented tree in Figure 11b. A SDE and a GA were also applied to the full N_5, and their parameter values have been fine-tuned. Values of $F = 0.3$ and $CR = 0.8$ were selected for the SDE, and the $P_c = 0.9$ and $P_m = 0.003$ were used for the GA.

[119] The sizes of the networks, the population sizes of the DE (including the SDE and the DE used in the proposed graph decomposition optimization method), and GA and the computational time for simulating each network are presented in Table 12. Values of $F = 0.5$ and $CR = 0.5$ were used for the DE applied to each subnetwork in the proposed method. As can be seen from Table 12, for the N_5 case study, the total computational runtime for hydraulically simulating each subnetwork 1000 times is 7.44 s, which is only 18% of that used by 1000 full network simulation. This indicates that the hydraulic simulation of the decomposed subnetworks is significantly faster than simulating the full network as a whole in terms of computational running time.

[121] For the N_5 case study, a total of 10 different runs with different starting random number seeds were performed for the proposed method, the SDE, and the GA. Figure 12 presents the solutions obtained by these three different optimization methods. The computational run time for each run of the proposed method has been converted to an equivalent number of full N_5 evaluations based on network simulation time in Table 12. The computational time used for identifying the 12 subnetworks is equivalent to 215 full N_5 evaluations.

[122] It may be clearly seen from Figure 12 that the proposed method after Step 4 was able to find lower cost solutions with significantly fewer number of full network evaluations compared to the SDE and GA. The optimal solutions found by the proposed graph decomposition optimization method after Step 3 are better than those obtained by the GA and comparable to those generated by the SDE but with significantly improved efficiency. Similar to that of

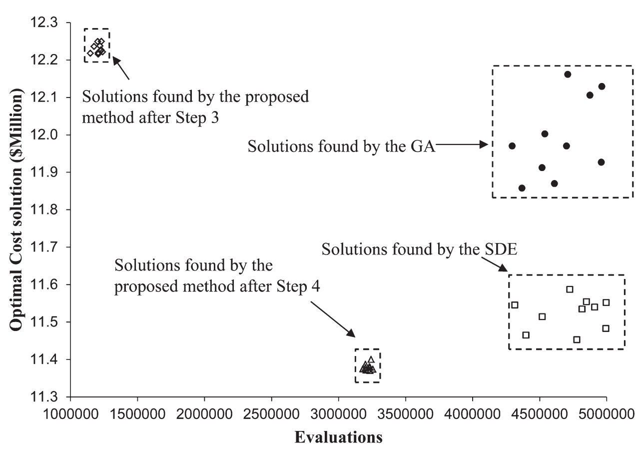

Figure 9. Solutions of the proposed method, the SDE, and the GA applied to N_4 case study.

Table 11. Algorithm Performance for the N_4 Case Study

Algorithm	Number of Different Runs	Best Solution Found in Millions ($)	Trials With Best Solution Found (%)	Average Cost Solution in Millions ($)	Average Number of Equivalent Evaluations to Find Best Solution
Proposed method[a]	10	12.22	0	12.23	1,208,324[b]
Proposed method[c]	10	11.37	30	11.38	3,215,685[b]
SDE[d]	10	11.45	0	11.52	4,730,200
GA[d]	10	11.85	0	11.99	4,654,000

[a]The results of the proposed method after the preconditioning subnetwork optimization (Step 3).
[b]The total computational overhead required by the proposed method has been converted to an equivalent number of whole network (N_4) evaluations.
[c]The results of the proposed method after the final subnetwork optimization (Step 4).
[d]Parameters were tuned.

the N_4 case study, the optimal solutions yielded by the proposed method for N_5 case study are closer to each other compared to the SDE and GA, showing greater robustness as similar cost solutions were found with different starting random number seeds.

[123] Table 13 presents the statistical results of the proposed method, the SDE, and the GA. The current best solution was found by the proposed method after Step 4 with a cost of $4.57 million, and this best solution was found eight times out of 10 runs with different random number seeds. The best solutions yielded by the SDE and GA were $4.60 and $4.72 million, respectively, which are 0.7% and 3.2% higher than the current best known solutions provided by the proposed method after Step 4. The proposed method exhibited the best performance in terms of comparing the efficiency to find optimal solutions as shown in Table 13. The average computational run time required by each run of the proposed method is equivalent to 2,720,668 full N_5 evaluations, which is 47% and 30% of those used by the SDE and GA.

[128] Interestingly, the proposed method after Step 3 was able to find lower cost solutions than the GA but with approximately five times the convergence speed. The best solutions found by the proposed method after Step 3 were only 0.2% higher than the best solution given by the SDE (the average costs of 10 solutions for both are the same as shown in Table 13), while the average number of evaluations required by the proposed method after Step 3 is only 21% of that used by the SDE.

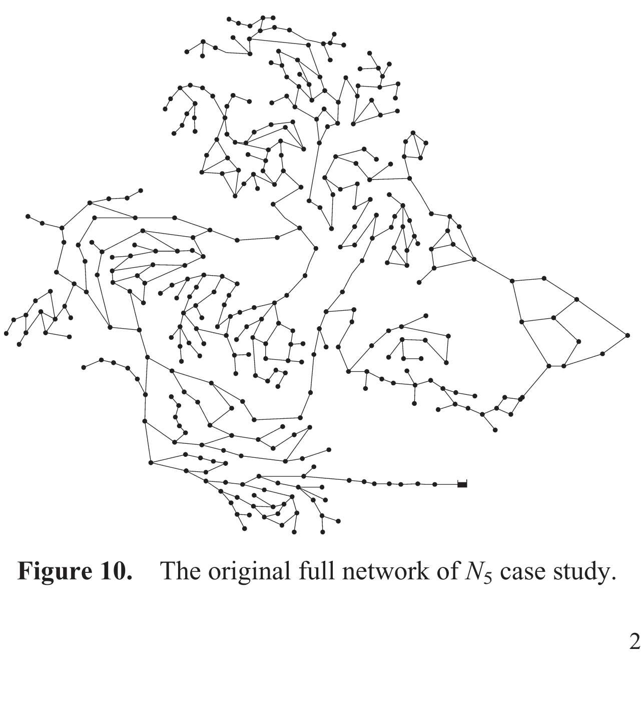

Figure 10. The original full network of N_5 case study.

5. Conclusion of Results and Future Work

[129] A novel optimization approach for WDS design has been developed and described in this paper. In the proposed method, a graph theory algorithm is employed to identify the subnetworks for the original full water network. The subnetworks, rather than the original full water network, are individually optimized by a DE in a predetermined sequence. Five case studies have been used to verify the effectiveness of the proposed method. A DE and a GA have also been applied to the full network for each case study (SDE) to enable a performance comparison with the proposed method.

[130] The results show that the proposed method is able to find the same lowest cost solution for the relatively small case study, while producing better optimal solutions for the relatively larger case studies than the SDE and GA. It was also noted that the proposed method was able to find extremely similar optimal solutions, if not identical, for each run with different starting random number seeds. This demonstrates the great robustness of the proposed method. In terms of efficiency, the proposed method significantly outperformed the SDE and GA for each case study.

[131] The proposed approach takes advantage of the fact that the EA (DE in this paper) is effective in exploring a relatively small search space. As the number of decision variables for each subnetwork is significantly less than the original whole network, the DE is able to exploit the substantially reduced search space quickly and effectively. This allows good quality optimal solutions for each subnetwork to be found with great efficiency.

[132] In spite of conducting multiple DE runs on each subnetwork, the total efficiency of the proposed method is still better than the SDE and GA. This can be attributed to the fact (i) the population size and the maximum allowable evaluations required by the DE applied to the subnetwork optimization were significantly smaller than the SDE applied to the original whole network and (ii) the computational time for simulating the subnetworks was considerably reduced compared to the original whole network.

[133] An important advantage of the proposed method is that, with multiple subnetworks in place, optimization of the water distribution systems can be undertaken using parallel computing technology. For the optimization of subnetworks at leaves and in the middle of the directed augmented tree, parallel computing technology can be

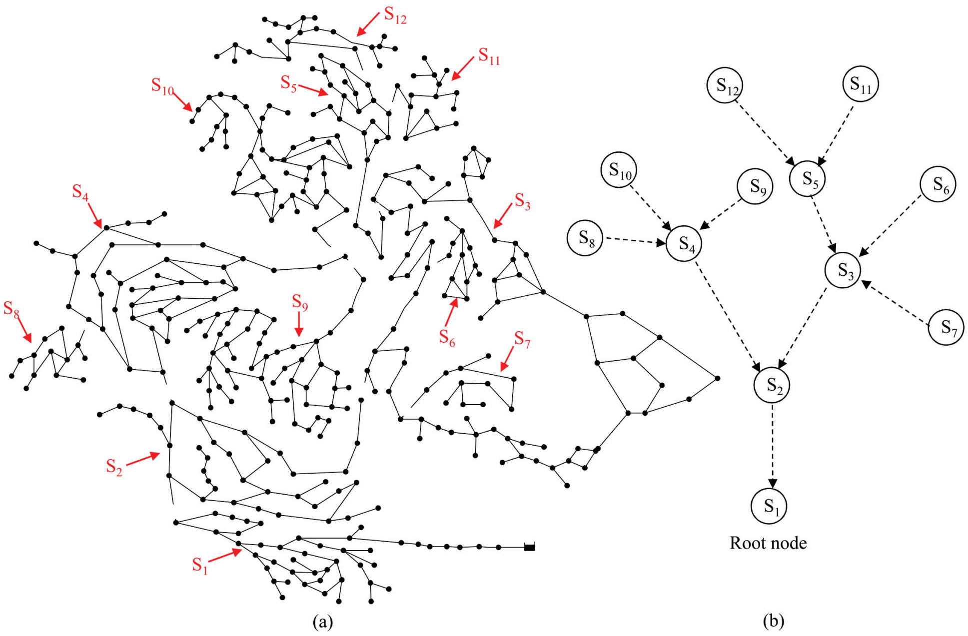

Figure 11. The subnetworks and the directed augmented tree of N_5. (a) The subnetworks. (b) The directed augmented tree.

Table 12. EA Parameter Values and the Hydraulic Simulation Time for Each Subnetwork and the Full N_5

EAs	Network	Number of Decision Variables and the Search Space Size	Population Size (N)	Maximum Number of Allowable Evaluations	The Computational Time for 1000 Simulations[a] (s)
SDE	N_5	433 (1.88×10^{496})	1000	10,000,000	42.06
GA	N_5	433 (1.88×10^{496})	1000	10,000,000	42.06
DE used in the proposed method	S_1	49 (1.44×10^{56})	200	200,000	0.72
	S_2	40 (7.00×10^{45})	200	200,000	0.61
	S_3	81 (6.86×10^{92})	200	500,000	2.13
	S_4	50 (2.02×10^{57})	200	200,000	0.81
	S_5	28 (1.23×10^{32})	100	100,000	0.30
	S_6	15 (1.56×10^{17})	50	50,000	0.23
	S_7	11 (4.05×10^{12})	50	50,000	0.14
	S_8	15 (1.56×10^{17})	50	50,000	0.23
	S_9	56 (1.52×10^{64})	200	200,000	0.92
	S_{10}	51 (2.83×10^{58})	200	200,000	0.74
	S_{11}	16 (2.18×10^{18})	50	50,000	0.22
	S_{12}	21 (1.17×10^{24})	100	100,000	0.27

[a]The 1000 simulations were based on randomly selected network configuration and conducted on the same computer configuration (Pentium PC (Inter R) at 3.0 GHz).

employed to conduct the optimization for different heads at the subnetwork cut nodes simultaneously. In addition, all the subnetworks at the leaves can also be optimized separately and simultaneously by parallel computing technology. As such, the efficiency of the whole optimization process can be massively improved in terms of computation time. This is a significant benefit when designing a real-world WDS, for which a large number of pipes and demand nodes are normally involved. It should be noted that the proposed method presented in this paper is not applicable to the networks for which subnetwork cut nodes do not exist (i.e., for networks that cannot be decomposed). However, it is very common for a water network to have multiple blocks and multiple trees in practice (that the network is decomposable), and the proposed method has advantages in efficiently finding good quality optimal solutions for this common type of network compared to other optimization methods as demonstrated in this paper.

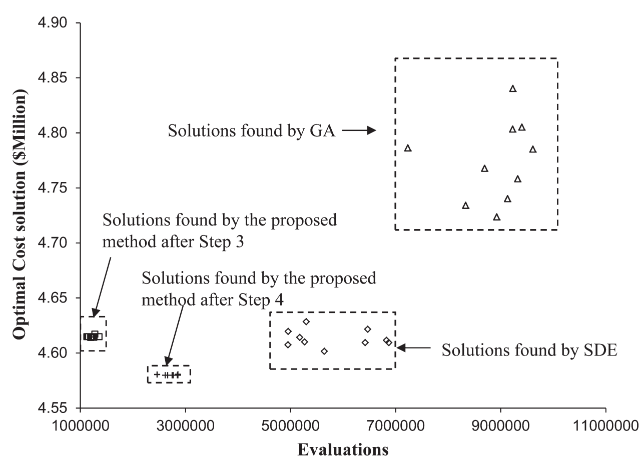

Figure 12. Solutions of the proposed method, the SDE, and the GA applied to N_5 case study.

Table 13. Algorithm Performance for the N_5 Case Study

Algorithm	Number of Different Runs	Best Solution Found in Millions ($)	Trials With Best Solution Found (%)	Average Cost Solution in Millions ($)	Average Number of Equivalent Evaluations to Find Best Solution
Proposed method[a]	10	4.61	0	4.61	1,220,924[b]
Proposed method[c]	10	4.57	80	4.58	2,720,668[b]
SDE[d]	10	4.60	0	4.61	5,786,300
GA[d]	10	4.72	0	4.77	8,909,500

[a]The results of the proposed graph decomposition optimization method after the preconditioning subnetwork optimization (Step 3).
[b]The total computational overhead required by the proposed method has been converted to an equivalent number of whole network (N_5) evaluations.
[c]The results of the proposed method after the final subnetwork optimization (Step 4).
[d]Parameters were tuned.

[134] The future research scope of the proposed method includes (i) applying the proposed method to more complex water networks that may include multiple reservoirs, pumps, valves, storage facilities, and pipes and (ii) extending the proposed method for multiobjective WDS optimization design.

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